

Monday 11 June 2012 – Afternoon

GCSE MATHEMATICS B

J567/03 Paper 3 (Higher Tier)

Solutions

Candidates answer on the Question Paper.

OCR supplied materials:
None

Duration: 1 hour 45 minutes

Other materials required:

- Geometrical instruments
- Tracing paper (optional)



Candidate forename		Candidate surname	
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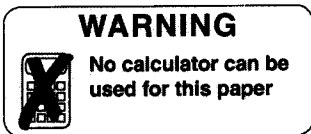
Centre number						Candidate number				
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INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the boxes above. Please write clearly and in capital letters.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Your answers should be supported with appropriate working. Marks may be given for a correct method even if the answer is incorrect.
- Write your answer to each question in the space provided. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.

INFORMATION FOR CANDIDATES

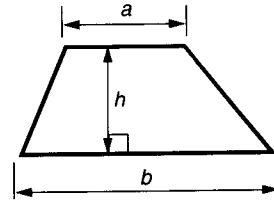
- The number of marks is given in brackets [] at the end of each question or part question.
- Your Quality of Written Communication is assessed in questions marked with an asterisk (*).
- The total number of marks for this paper is **100**.
- This document consists of **20** pages. Any blank pages are indicated.



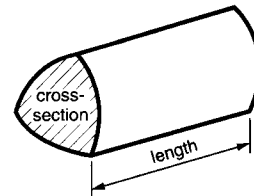
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Formulae Sheet: Higher Tier

Area of trapezium = $\frac{1}{2}(a + b)h$



Volume of prism = (area of cross-section) \times length

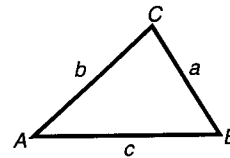


In any triangle ABC

Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

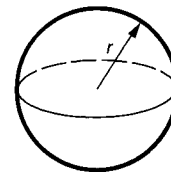
Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle = $\frac{1}{2} ab \sin C$



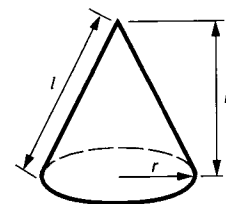
Volume of sphere = $\frac{4}{3} \pi r^3$

Surface area of sphere = $4\pi r^2$



Volume of cone = $\frac{1}{3} \pi r^2 h$

Curved surface area of cone = $\pi r l$



The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$,
where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

PLEASE DO NOT WRITE ON THIS PAGE

1 (a) Write as a decimal.

$$(i) \frac{3}{50} = \frac{6}{100} = 0.06$$

$$(a)(i) \underline{\hspace{10em} 0.06 \hspace{10em}} [1]$$

$$(ii) \frac{2}{9} \quad 9 \overline{) 2.222 \dots} = 0.\dot{2}$$

$$(ii) \underline{\hspace{10em} 0.\dot{2} \hspace{10em}} [1]$$

(b) Work out.

$$(i) \frac{5^2 \times 5^5}{5^4} = \frac{5^7}{5^4} = 5^3 = 125$$

$$(b)(i) \underline{\hspace{10em} 125 \hspace{10em}} [2]$$

$$(ii) 3\frac{1}{3} - 1\frac{5}{6}$$

Give your answer in its simplest form.

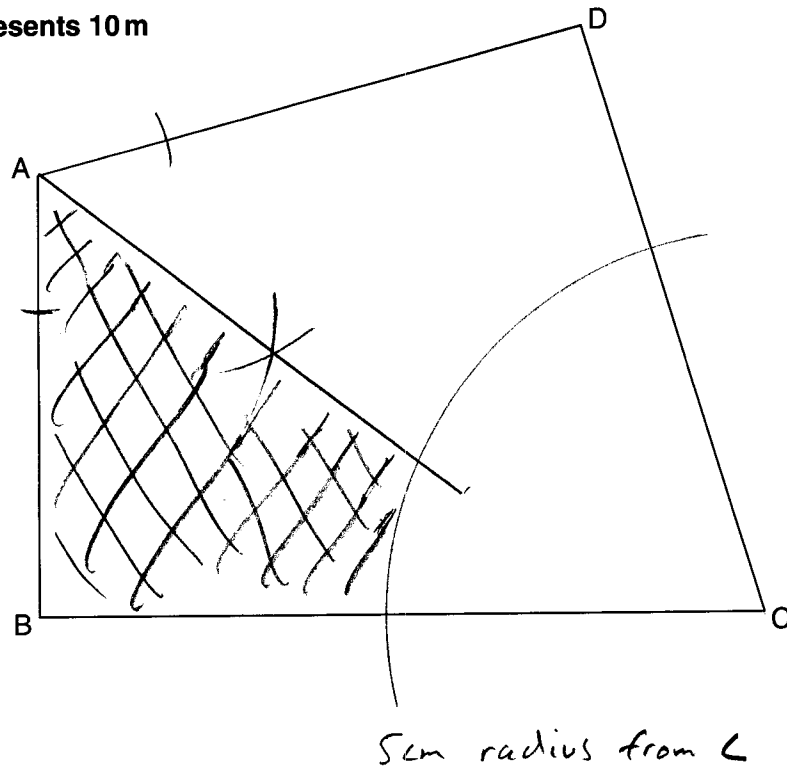
$$= 1\cancel{2} \frac{2-5}{6} = 1\frac{3}{6} = 1\frac{1}{2}$$

$$(ii) \underline{\hspace{10em} \hspace{10em} [3]$$

2 The scale drawing shows a field ABCD.

Scale: 1 cm represents 10 m

Bisect $\angle BAD$



Tom pitches his tent in the field.

The tent is pitched

- closer to AB than to AD
- more than 50 m from C.

Construct and shade the region where Tom's tent could be pitched.
Leave in all your construction lines.

[4]

- 3 Nita is making a fruit drink.
She mixes apple juice and mango juice in the ratio 3 : 1.

- (a) How much of each type of juice will she need to make 1 litre of the fruit drink?
Give your answers in millilitres.

$$3 + 1 = 4 \text{ parts}$$

$$4 \text{ parts} = 1000 \text{ ml}$$

$$1 \text{ part} = 250 \text{ ml}$$

$$\begin{aligned} \text{Apple } 3 \times 250 &= 750 \\ \text{Mango } 1 \times 250 &= 250 \end{aligned}$$

(a) Apple juice 750 ml
Mango juice 250 ml [2]

- (b)* Apple juice costs 56p for a 1-litre carton.
Mango juice costs £1.20 for a 1-litre carton.
A pack of 80 plastic cups costs £1.

Nita sells her fruit drink at a school concert in 250 ml cups for 60p each.
She gives all the **profit** she makes to the school fund.
Nita makes 80 cups of the fruit drink and sells them all.

How much money does she give to the school fund?

$$\underline{\text{Takings}} \quad 80 \times 60 = 4800p = \pounds 48.00$$

Costs

$$250 \text{ ml} = \frac{1}{4} \text{ litre}$$

$$\Rightarrow 80 \text{ cups} = 80 \times \frac{1}{4} = 20 \text{ litres}$$

$$\Rightarrow 15 \text{ litres apple and } 5 \text{ litres mango}$$

$$\begin{array}{l} \text{Apple} \quad 15 \times 56 = 560 + 280 = 840p = \pounds 8.40 \\ \text{Mango} \quad 5 \times 120 = 600p = \pounds 6.00 \\ \text{Pack of cups} \quad = \pounds 1.00 \\ \hline \pounds 15.40 \end{array}$$

$$\text{Profit} = \text{Takings} - \text{Costs}$$

$$\begin{aligned} &= \pounds 48.00 \\ &\quad \pounds 15.40 - \\ &\hline &\pounds 32.60 \end{aligned}$$

$$(b) \quad \underline{\pounds 32.60} \quad [5]$$

- 4 (a) This two-way table summarises some information about the membership of Sheldon Running Club.

	Age under 18	Age 18–59	Age 60+
Male	30	68	22
Female	14	46	20

- (i) How many of the members are female?

$$14 + 46 + 20 = 80$$

(a)(i) 80 [1]

- (ii) One of the males is selected at random.

Find the probability that he is under 18.

$$\text{Males } 30 + 68 + 22 = 120$$

$$\text{Prob(Under 18)} = \frac{30}{120} = \frac{1}{4}$$

(ii) $\frac{1}{4}$ [2]

- (iii) What percentage of the whole membership is aged 18–59?

$$\text{Total membership } 80 + 120 = 200$$

$$18-59 \rightarrow 68 + 46 = 114$$

$$\frac{114}{200} \times 100 = 57\%$$

(iii) 57 % [2]

- (b) Twenty members of Sheldon Running Club run a 10 km race. Their times, in minutes, are listed below.

63 40 42 54 52 37 58 72 60 45
39 46 65 55 48 51 45 62 51 57

- (i) Complete this stem and leaf diagram to represent the data.

3 | 7 9
4 | 0 2 5 6 8 5
5 | 4 2 8 5 1 1 7
6 | 3 0 5 2
7 | 2

3	7 9
4	0 2 5 5 6 8
5	1 1 2 4 5 7 8
6	0 2 3 5
7	2

Key: 6 | 3 represents 63 minutes

[3]

- (ii) Find the range of the times. $\text{Range} = 72 - 37 = 35$

(b)(ii) 35 minutes [1]

- (iii) Joe says

Half of these runners ran at an average speed of **more than** 12 km/h.

Is he correct? Show how you decide.

12 km/h \rightarrow 1 km in 5 mins
 \rightarrow 10 km in 50 mins

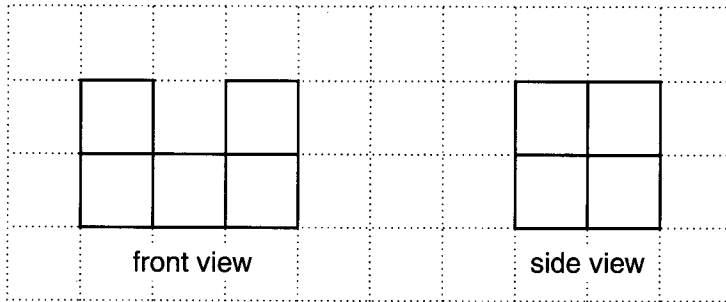
How many ran in under 50 mins Answer: 8

8 out of 20 is less than half the runners

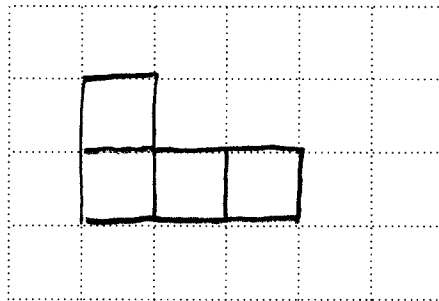
No because only 8 out of 20 (ie 40%)
ran faster than 12 km/h [4]

5 Alec has 6 one-centimetre cubes.

- (a) Alec makes a solid with the six cubes.
Its front view and side view are shown below.



On the grid below, draw the plan view of the solid.

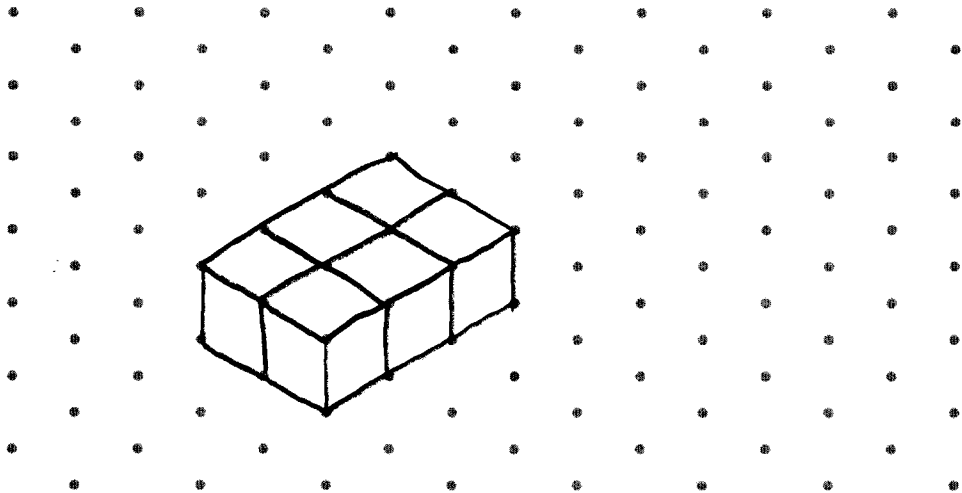


or any of these

[2]

- (b) Alec rearranges the 6 cubes to make a cuboid with the **smallest** possible surface area.

(i) On the isometric grid below, sketch this cuboid.

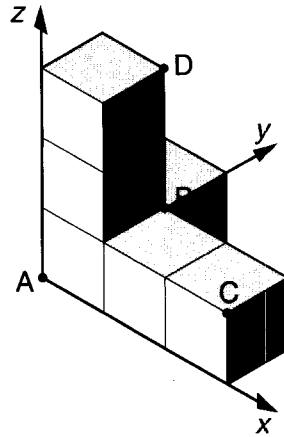


[2]

(ii) Find the total surface area of this cuboid.

(b)(ii) _____ cm² [2]

- (c) Alec makes a different solid with the 6 cubes. He places the solid on the axes below.



The coordinates of point A are $(0, 0, 0)$ and the coordinates of point B are $(1, 1, 1)$.

Write down the coordinates of

- (i) point C,

(c)(i) $(\underline{3}, \underline{0}, \underline{1})$ [1]

- (ii) point D.

(ii) $(\underline{1}, \underline{1}, \underline{3})$ [1]

- 6 (a) Here are the first four terms of a sequence.

8 11 14 17

Write an expression for the n th term of this sequence.

(a) $\underline{3n + 5}$ [2]

- (b) The n th term of another sequence is given by $12 - 5n$.

Write down the first three terms of this sequence.

$$\begin{aligned} 12 - 5 \times 1 &= 12 - 5 = 7 \\ 12 - 5 \times 2 &= 12 - 10 = 2 \\ 12 - 5 \times 3 &= 12 - 15 = -3 \end{aligned}$$

(b) $\underline{7}, \underline{2}, \underline{-3}$ [2]

7 (a) Expand.

$$3(a + 4)$$

$$(a) \underline{3a + 12} \quad [1]$$

(b) Factorise fully.

$$2b^2 + 6b$$

$$(b) \underline{2b(b + 3)} \quad [2]$$

(c) Solve.

$$5x - 2 = 3x + 7$$

$$5x - 3x = 7 + 2$$

$$2x = 9$$

$$x = \frac{9}{2}$$

$$x = 4\frac{1}{2}$$

$$(c) x = \underline{4\frac{1}{2}} \quad [3]$$

(d) (i) Solve this inequality.

$$4y - 3 \geq 5$$

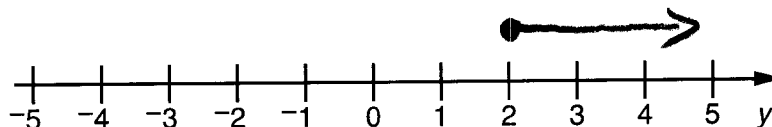
$$4y \geq 5 + 3$$

$$4y \geq 8$$

$$y \geq \frac{8}{4}$$

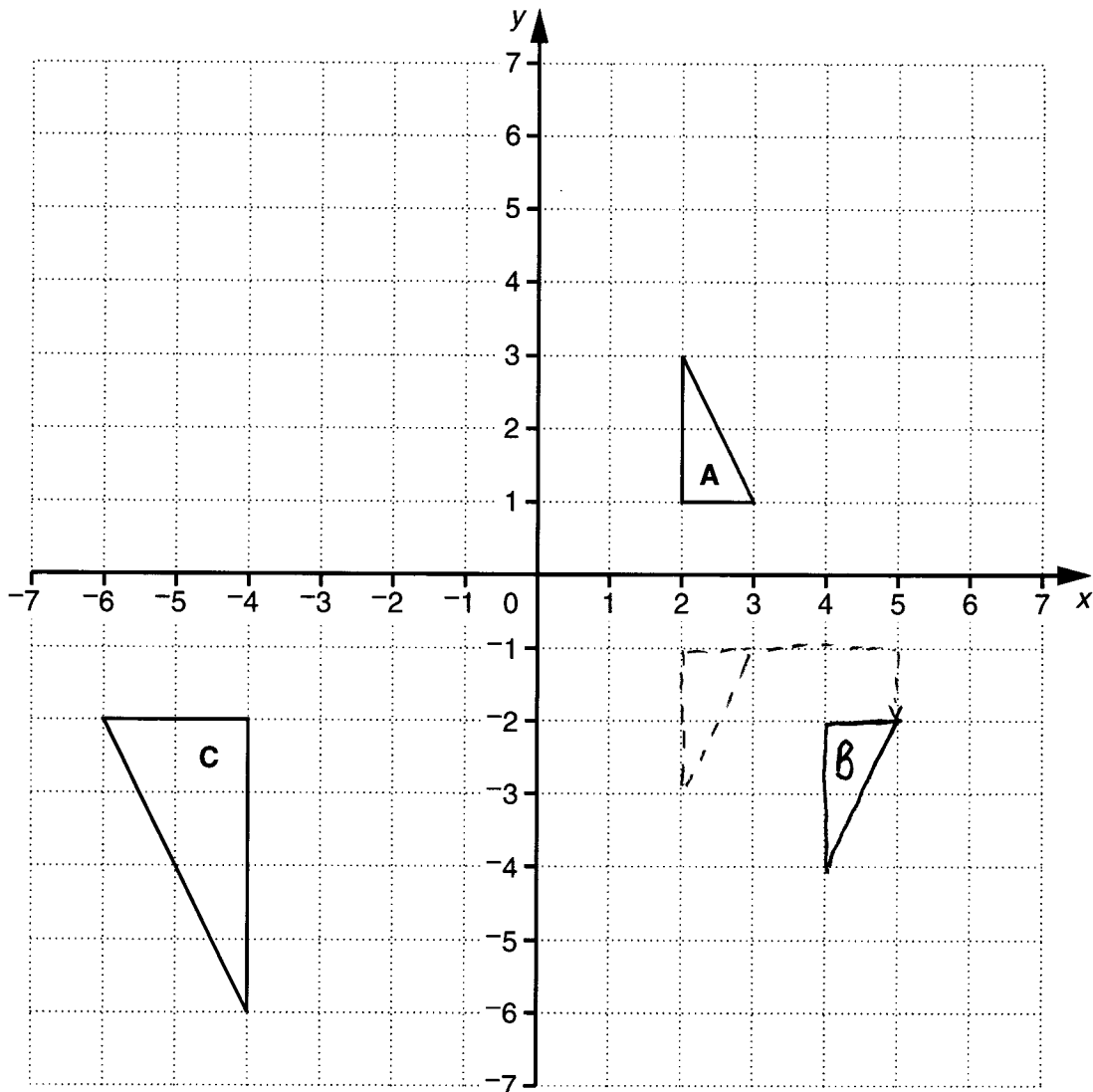
$$(d)(i) \underline{y \geq 2} \quad [2]$$

(ii) Represent your solution to part (d)(i) on the number line.



[1]

8 Triangles **A** and **C** are drawn on the grid below.



(a) Triangle **B** is the image of triangle **A** after

- a reflection in the x -axis, followed by
- a translation of $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$.

Draw and label triangle **B** on the grid.

[3]

(b) Describe fully the **single** transformation that maps triangle **A** onto triangle **C**.

Enlargement by scale factor -2 about
centre of enlargement $(0,0)$

[3]

9 (a) The population of the UK in 2008 was 61.4 million.

(i) Write 61.4 million in standard form.

$$= 61,400,000$$

$$(a)(i) \underline{6.14 \times 10^7} \quad [1]$$

(ii) The budget for building the facilities for the London 2012 Olympics was £9325 million. Chloe estimates that this was about £1500 per person in the UK.

Show that Chloe's estimate is **not** correct.

[2]

$$\begin{aligned} \text{Amount per person} &\approx \frac{9325 \text{ million}}{61.4 \text{ million}} \\ &\approx \frac{9000}{60} \\ &= \pounds 150 \end{aligned}$$

She has overestimated by a factor of 10

(b) The mass of concrete used in building the Olympic Park was 698 792 tonnes. The mass of concrete used in building the Olympic Village was 648 500 tonnes.

Find the total mass of concrete used to build the Park and the Village. Give your answer in standard form, correct to three significant figures.

$$\begin{array}{r} 698,792 \\ 648,500 + \\ \hline 1347,292 \end{array} = 1.35 \times 10^6 \text{ tonnes}$$

to 3 s.f.

$$(b) \underline{1.35 \times 10^6} \text{ tonnes} \quad [2]$$

- 11 A ball is dropped from the top of a cliff into the sea.
The height, h metres, of the ball above sea level, t seconds after it is dropped, is given by

$$h = 90 - 5t^2.$$

When $t = 0$, $h = 90$

- (a) Write down the height of the top of the cliff above sea level.

(a) 90 m [1]

- (b) Complete this table of values for the equation $h = 90 - 5t^2$.

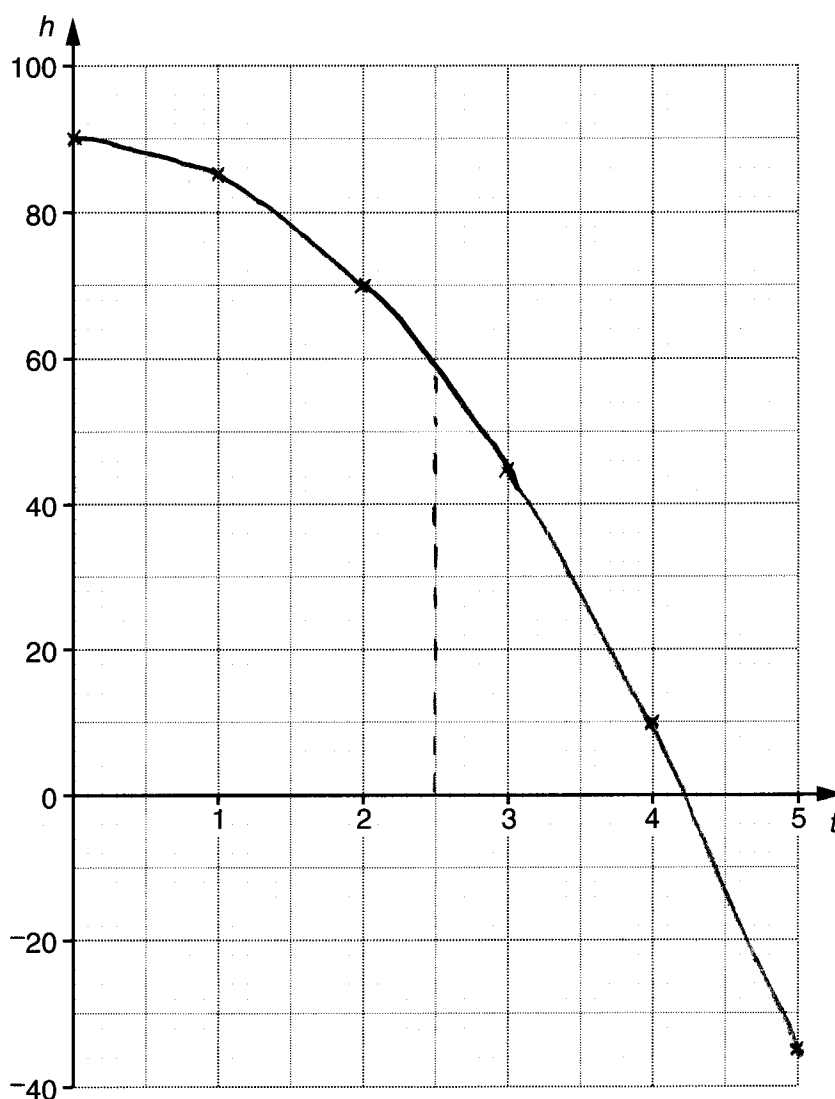
$$90 - 5 \times 1^2 = 90 - 5 = 85$$

$$90 - 5 \times 2^2 = 90 - 20 = 70$$

t	0	1	2	3	4	5
h	90	85	70	45	10	-35

[2]

- (c) On the axes below draw the graph of $h = 90 - 5t^2$, for values of t from 0 to 5.



[2]

(d) Use your graph to find

(i) the height of the ball above sea level after 2.5 seconds,

(d)(i) 58 m [1]

(ii) the time taken for the ball to reach sea level.

(ii) 4.2 s [1]

12 In a football league, teams gain 3 points for a win and 1 point for a draw.

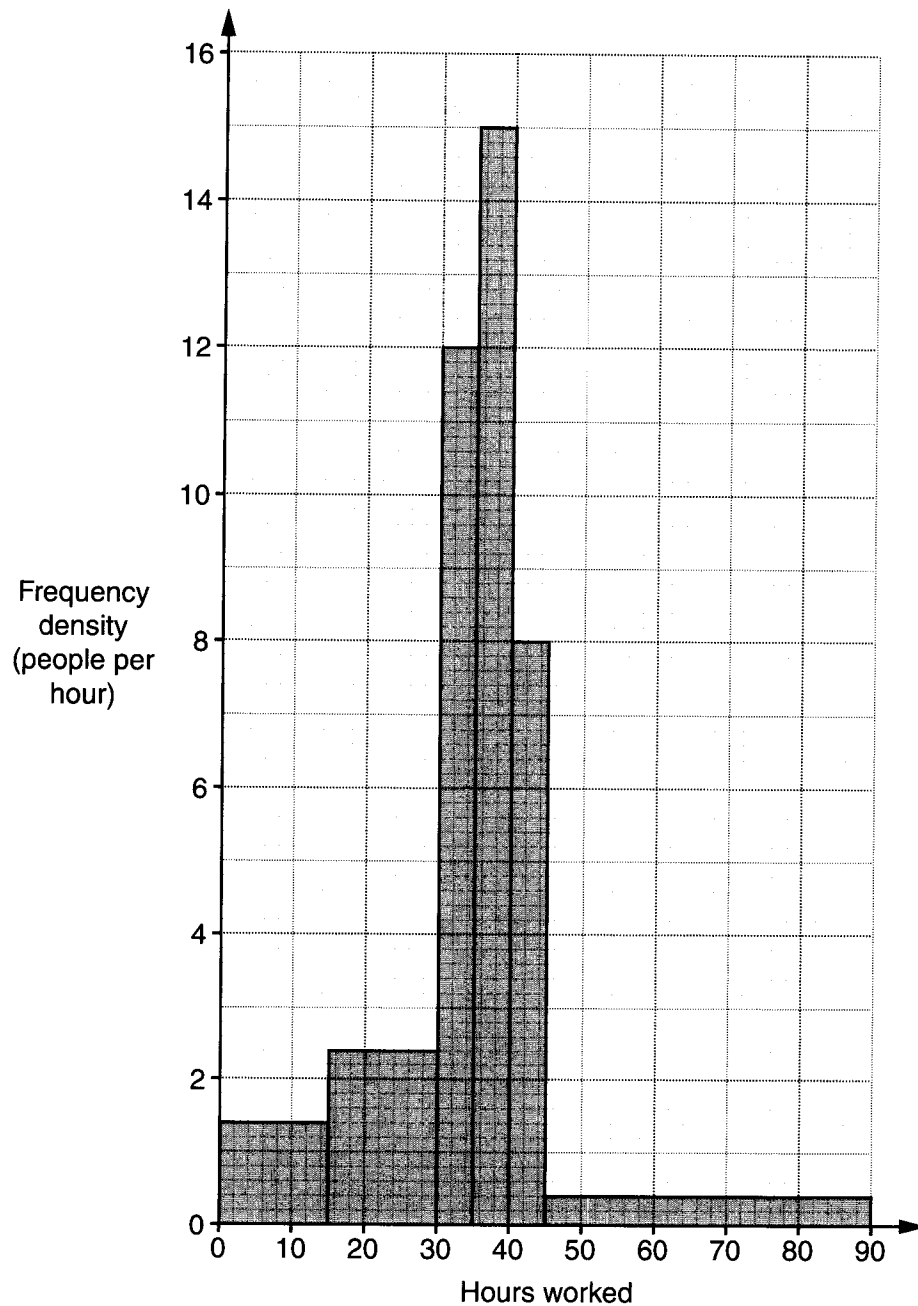
East Hill have either won or drawn each of their last 12 matches.
They gained a total of 28 points from these matches.

Find the number of these matches that East Hill have won.

$$\begin{array}{l}
 7 \text{ Wins} \quad 5 \text{ Draws} \quad \rightarrow \quad 7 \times 3 + 5 \times 1 = 26 \text{ points} \\
 8 \text{ Wins} \quad 4 \text{ Draws} \quad \rightarrow \quad 8 \times 3 + 4 \times 1 = 28 \text{ points} \checkmark \\
 9 \text{ Wins} \quad 3 \text{ Draws} \quad \rightarrow \quad 9 \times 3 + 3 \times 1 = 30 \text{ points}
 \end{array}$$

8 wins [3]

- 13 Karen surveys a group of people to find out how many hours, in total, they had worked in the previous week. Her results are summarised in the histogram.



- (a) How many people worked between 15 and 30 hours in the previous week?

$$\text{Frequency} = \text{Freq Density} \times \text{Width}$$

$$= 2.4 \times 15 = 36$$

(a) 36 [1]

- (b) A national survey found that 54% of people had an average working week of between 30 and 45 hours.

How do the results of Karen's survey compare with the national survey?

Find Karen's sample size by calculating sum of frequencies

$$\begin{array}{r}
 1.4 \times 15 = 21 \\
 2.4 \times 15 = 36 \\
 12 \times 5 = 60 \\
 15 \times 5 = 75 \\
 8 \times 5 = 40 \\
 0.4 \times 45 = 18 \\
 \hline
 250
 \end{array}$$

$$\begin{array}{l}
 30-45 \text{ hrs people} \\
 = 60 + 75 + 40 \\
 = 175 \\
 \text{Proportion in Karen's survey} \\
 = \frac{175}{250} \times 100 \\
 = \frac{7}{10} \times 100 = 70\%
 \end{array}$$

This is considerably more than in the National Survey.

[4]

- 14 The frequency, F hertz, of the note produced by a vibrating string is inversely proportional to its length, d centimetres.

When the length of the string is 60 cm, the frequency of the note is 300 hertz.

- (a) Find an equation connecting F and d .

$$F = \frac{k}{d}$$

$$k = 18000$$

When $d = 60$, $F = 300$

$$300 = \frac{k}{60}$$

$$\Rightarrow 300 \times 60 = k$$

$$F = \frac{18000}{d}$$

(a) _____ [2]

- (b) Find the frequency of the note produced when the string is 40 cm long.

When $d = 40$

$$F = \frac{18000}{40} = \frac{1800}{4} = 450$$

(b) 450 hertz [1]

- 15 (a) Write $x^2 + 8x + 10$ in the form $(x + a)^2 + b$.

$$= (x + 4)^2 + 10 - 16$$

$$= (x + 4)^2 - 6$$

(a) $(x + 4)^2 - 6$ [3]

- (b) Hence state the minimum value of $x^2 + 8x + 10$.

occurs when $x = -4$ and $(x + 4)^2 = 0$

(b) -6 [1]

16 Vector $\mathbf{p} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$ and vector $\mathbf{q} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$.

(a) Calculate.

$$(i) \quad 3\mathbf{p} = 3 \begin{pmatrix} 3 \\ -1 \end{pmatrix} = \begin{pmatrix} 9 \\ -3 \end{pmatrix}$$

(a)(i)

$$\begin{pmatrix} 9 \\ -3 \end{pmatrix}$$

[1]

(ii) $\mathbf{p} + \mathbf{q}$

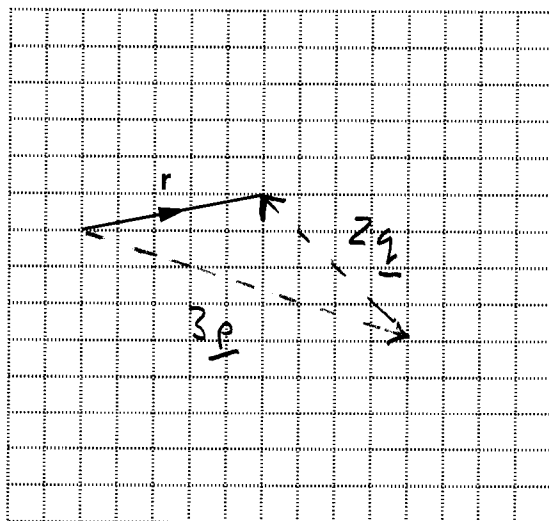
$$\begin{pmatrix} 3 \\ -1 \end{pmatrix} + \begin{pmatrix} -2 \\ 2 \end{pmatrix} = \begin{pmatrix} 3-2 \\ -1+2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

(ii)

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

[1]

(b) The vector \mathbf{r} is shown on the grid below.



Given that $\mathbf{r} = j\mathbf{p} + k\mathbf{q}$, find the values of j and k .

$$(b) \quad j = \underline{3} \quad k = \underline{2} \quad [2]$$

TURN OVER FOR QUESTIONS 17 AND 18

