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RECOGNISING ACHIEVEMENT

H

GENERAL CERTIFICATE OF SECONDARY EDUCATION

MATHEMATICS SYLLABUS A

J512/03

Paper 3 (Higher Tier)

Solutions

OCE/10494

Candidates answer on the Question Paper

OCR Supplied Materials:
None

Other Materials Required:
- Geometrical instruments
- Tracing paper (optional)

Monday 7 June 2010
Afternoon

Duration: 2 hours

J512 03

Candidate Forename		Candidate Surname	

Centre Number						Candidate Number				

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the boxes above.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Show your working. Marks may be given for a correct method even if the answer is incorrect.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Write your answer to each question in the space provided. Additional paper may be used if necessary but you must clearly show your Candidate Number, Centre Number and question number(s).

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- The total number of marks for this paper is **100**.
- This document consists of **24** pages. Any blank pages are indicated.

WARNING

No calculator can be used for this paper

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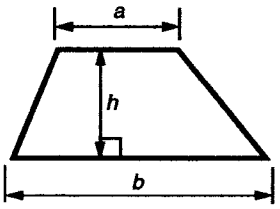
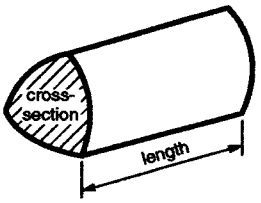
Formulae Sheet: Higher Tier

Area of trapezium $= \frac{1}{2}(a + b)h$



Volume of prism = (area of cross-section) × length

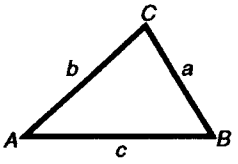


In any triangle *ABC*

Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

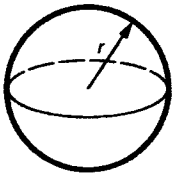
Cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$

Area of triangle $= \frac{1}{2}ab\sin C$



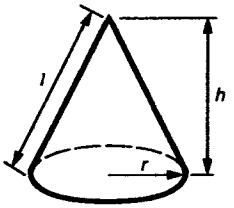
Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$



Volume of cone $= \frac{1}{3}\pi r^2 h$

Curved surface area of cone $= \pi rl$



The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$,
where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

PLEASE DO NOT WRITE ON THIS PAGE

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1 **(a)** Explain why each answer is incorrect.

(i) $3.7 \times -4.5 = 16.65$

A positive multiplied by a negative gives a negative answer

[1]

(ii) $\sqrt{67.24} = 7.2$

$\sqrt{64} = 8$ so answer should be greater than 8

[1]

(iii) $6.3 \div 0.9 = 70$

$6.3 \div 0.9$

$= 63 \div 9 = 7$

[1]

(b) Work out.

(i) $(16 + 5) \div 3 = 21 \div 3 = 7$

(b)(i) 7 [1]

(ii) $4 + 6 \times 3 = 4 + 18 = 22$

(ii) 22 [1]

(c) Put one pair of brackets into this equation to make it correct.

$$44 - 26 - (3 + 8) = 7$$

[1]

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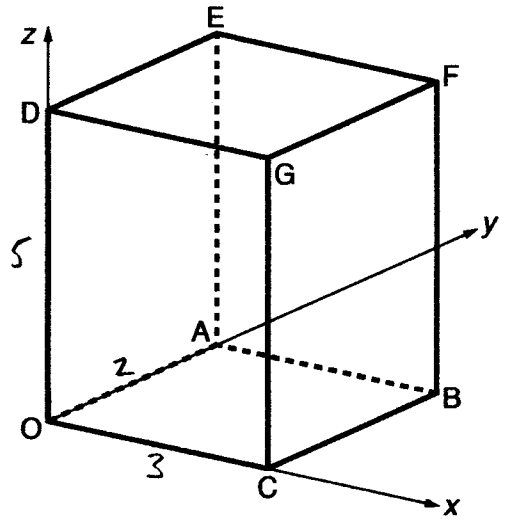
2 In the diagram, O is the origin.
Each line is parallel to one of the axes.
OA = 2, OC = 3 and OD = 5 units.



Write down the coordinates of

(a) D,

(a) (0 , 0 , 5) **[1]**

(b) F,

(b) (3 , 2 , 5) **[1]**

(c) the midpoint of AB.

(c) (1·5 , 2 , 0) **[1]**

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3 Solve.

$2(x + 7) = 10$

$2x + 14 = 10$

$2x = 10 - 14$

$2x = -4$

$x = \frac{-4}{2}$

$x = -2$

$x = -2$ **[3]**

4 60% of the members of a youth club are girls.
There are 12 boys in the youth club.

How many members are there in the youth club altogether?

40% are boys

$40\% = 12$

$10\% = \frac{12}{4} = 3$

$100\% = 3 \times 10 = 30$

30 **[4]**

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5 **(a)** Work out the value of $x^2 + 5x$ when

(i) $x = -2$, $(-2)^2 + 5(-2)$

$= +4 - 10 = -6$

(a)(i) -6 **[2]**

(ii) $x = \frac{1}{2}$. $(\frac{1}{2})^2 + 5(\frac{1}{2})$

$= \frac{1}{4} + \frac{5}{2}$

$= \frac{1}{4} + \frac{10}{4} = \frac{11}{4} = 2\frac{3}{4}$ **(ii)** $2\frac{3}{4}$ **[2]**

(b) The formula for the nth term of a number sequence is $3n + 2$.

Work out the **first three** terms of this sequence.

$3(1)+2 = 3+2 = 5$

$3(2)+2 = 6+2 = 8$

$3(3)+2 = 9+2 = 11$

(b) 5, 8, 11 **[2]**

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6 A biased spinner is numbered 1, 2, 3 and 4.
The table shows the probability of the spinner landing on each of the numbers.

Number	1	2	3	4
Probability	0.25	0.4	0.15	0.2

(a) Work out the probability that the spinner lands on 3 or 4 on the next spin.

P(3or4) = 0·15 + 0·2 = 0·35

(a) 0·35 **[2]**

(b) The spinner is spun twice.

Work out the probability that the spinner lands on 2 each time.

0.4 x 0.4 = 0·16

(b) 0·16 **[2]**

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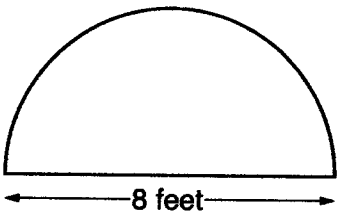
7 *In this question, take the value of π to be 3.*

Emma visited a stately home.
In one of the rooms there was a semi-circular carpet.
The diameter of the carpet was 8 feet.



Work out the area of the carpet.
Give the units of your answer.

radius = 4 ft

Area of circle = πr^2

Area of semi-circle = $\frac{\pi r^2}{2} = \frac{3 \times 4^2}{2} = \frac{3 \times 16}{2} = 24$

24 ft^2 [3]

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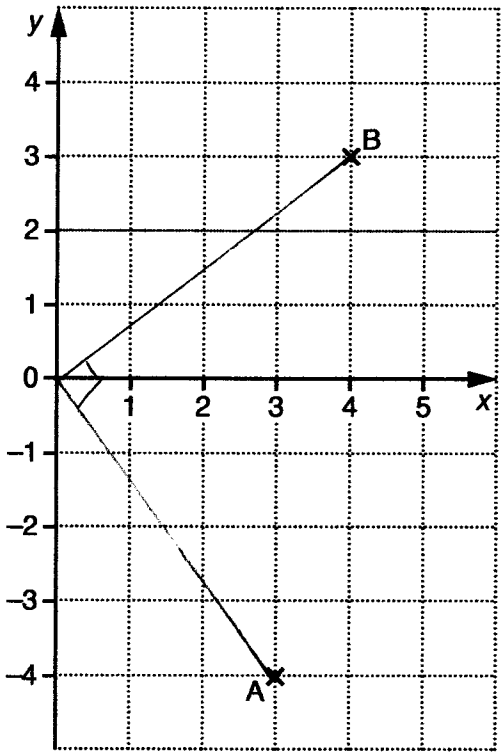
8



(a) An anticlockwise rotation, centre (0, 0), will map point A onto point B.

What is the angle of the rotation?

(a) 90° ° **[1]**

(b) Describe **fully** a different type of transformation which will map point A onto point B.

Translation by $\begin{pmatrix} 1 \\ 7 \end{pmatrix}$

[3]

(c) Point A can be mapped onto point B by a reflection in a line parallel to the *x*-axis followed by a reflection in a line parallel to the *y*-axis.

Write down the equation of each of these lines.

.....

(c) $y = -\frac{1}{2}$ and $x = 3\frac{1}{2}$ **[2]**

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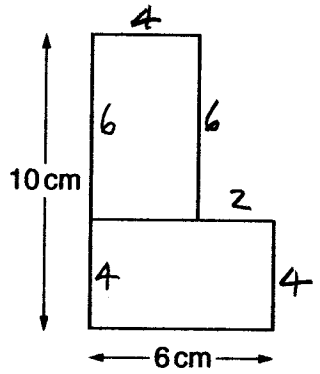
9 **(a)** The diagram shows two **identical** rectangles joined to make an L shape.



NOT TO SCALE

Each rectangle must be 6cm by 4cm

Work out

(i) the total area of the L shape,

$6 \times 4 = 24$

$6 \times 4 = 24$ +

$48\ cm^2$

(a)(i) 48 cm² **[3]**

(ii) the perimeter of the L shape.

$4 + 6 + 2 + 4 + 6 + 10 = 32\ cm$

(ii) 32 cm **[3]**

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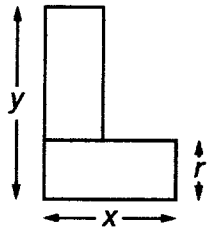
(b) Two other identical rectangles are joined to make an L shape.



(i) Work out the length marked *r*.
Give your answer in terms of *x* and *y*.

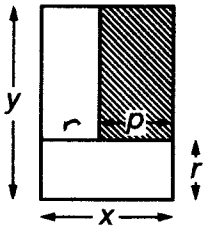
.....

(b)(i) $y - x$ **[1]**

(ii) Use your answer to part **(b)(i)** to explain why $y > x$.

$y > x$ because *r is a positive number representing an actual length* **[1]**

The L shape is enclosed within an outer rectangle.



(iii) Work out the length marked *p*.
Give your answer in terms of *x* and *y*.

$p = x - r$
$= x - (y - x)$
$= x - y + x$
$= 2x - y$

.....

(iii) _____ **[1]**

(iv) Use your answer to part **(b)(iii)** to explain why $y < 2x$.

$y < 2x$ because *p is a positive number representing an actual length* **[1]**

(v) Work out the shaded area as a fraction of the area of the outer rectangle.
Give your answer in terms of *x* and *y*.

Shaded area $= px = (2x - y)x$

.....

Fraction is $\frac{(2x-y)x}{xy} = \frac{2x-y}{y}$

(v) _____ **[2]**

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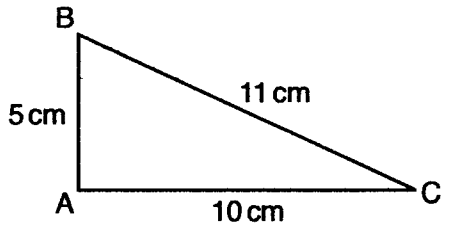
10 Triangle ABC has sides of length 5 cm, 10 cm and 11 cm.



NOT TO SCALE

(a) Show by calculation that angle A is **not** a right angle.

If $A = 90°$ then by Pythagoras

$5^2 + 10^2 = 11^2$

$25 + 100 = 121$

$125 = 121$ ✗

∴ A is not a right angle [3]

(b) Is angle A greater than or less than 90°?

Use your calculation in part **(a)** to support your decision.

Angle A is less than 90° because BC is less than $\sqrt{125}$ or 11 is less than $\sqrt{125}$ [2]

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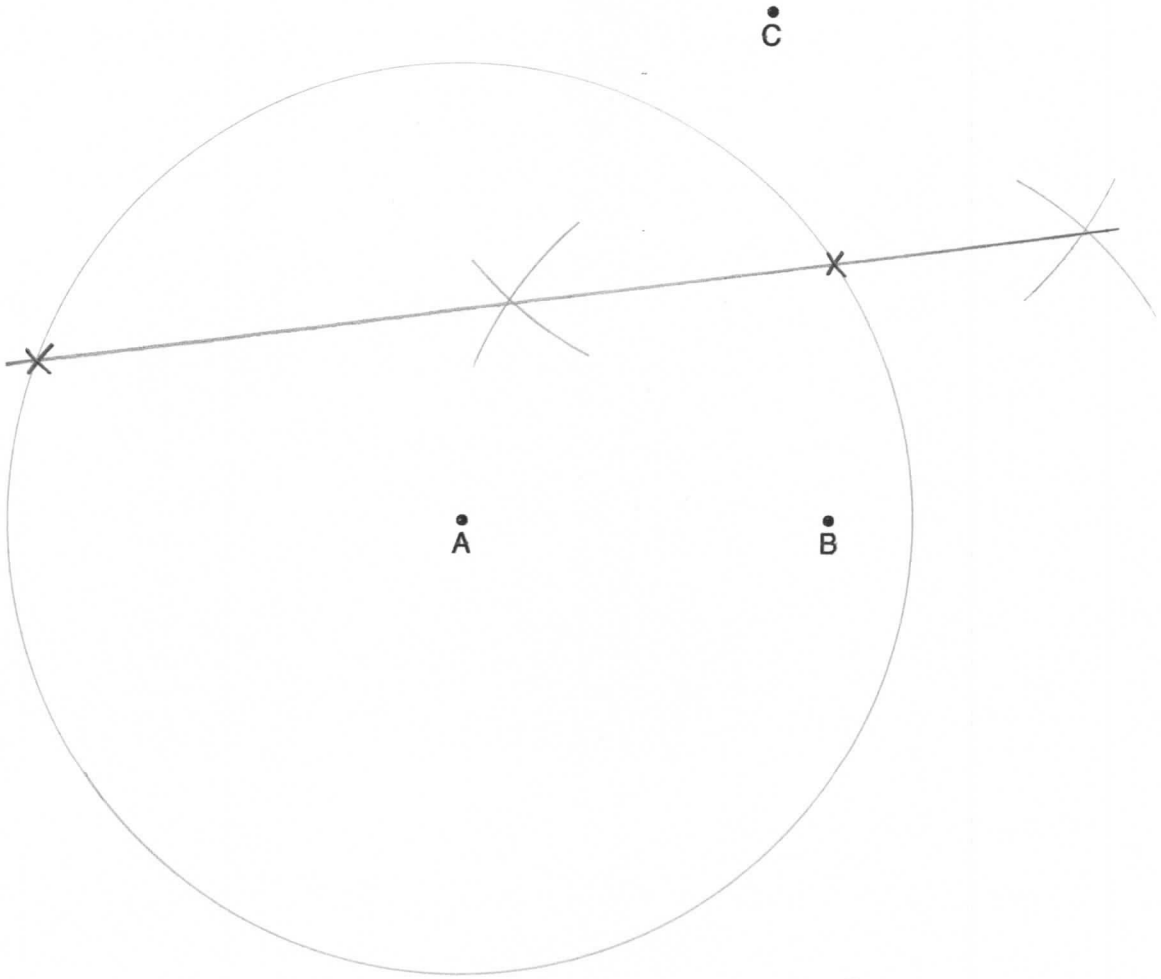
11 *Use ruler and compasses only in this question.*

Find and indicate clearly all possible points that are both

- 6 cm from A

and

- equidistant from B and C.



[5]

Two points marked by x

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12 Work out.

$$2\frac{2}{3} \times 1\frac{1}{7}$$

Give your answer as a mixed number.

$= \frac{8}{3} \times \frac{8}{7} = \frac{64}{21} = 3\frac{1}{21}$

$3\frac{1}{21}$ **[3]**

13 **(a)** Factorise.

$5x^2 - 10xy$ $= 5x(x - 2y)$

(a) $5x(x - 2y)$ **[2]**

(b) Rearrange this formula to make h the subject.

$A = 2\pi r(r + h)$

$A = 2\pi r^2 + 2\pi rh$

$A - 2\pi r^2 = 2\pi rh$

$\frac{A - 2\pi r^2}{2\pi r} = h$

(b) $h = \frac{A - 2\pi r^2}{2\pi r}$ **[3]**

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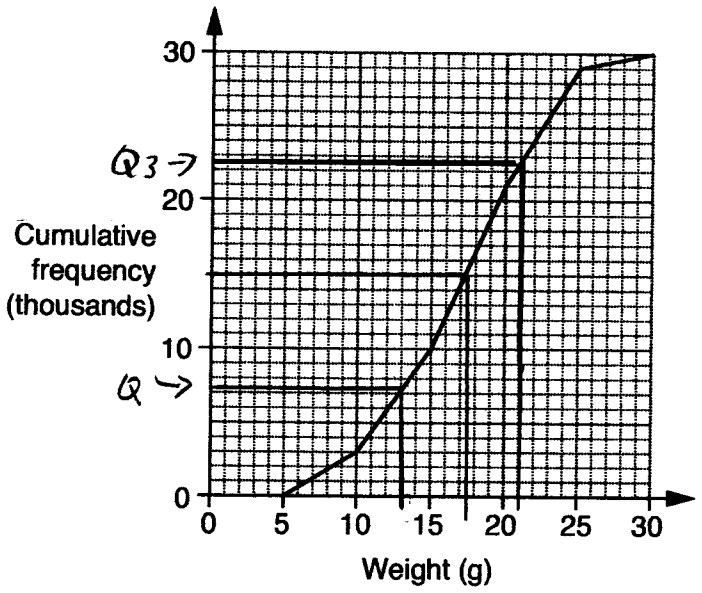
14 A farmer grows strawberries.

(a) The cumulative frequency diagram shows the distribution of the weights of thirty thousand strawberries picked one day.



Use the graph to find

(i) the median weight of the strawberries,

(a)(i) 17.5 g **[1]**

(ii) the interquartile range of the weights,

$Q_3 - Q_1 = 21 - 13 = 8$

(ii) 8 g **[2]**

(iii) the number of strawberries weighing over 20 g.

21000 strawberries up to 20g

so 30000 − 21000 strawberries over 20g

= 9000

(iii) 9000 **[2]**

(b) One of the strawberries weighs 12 g, correct to the nearest gram.

What is the upper and lower bound of this weight?

(b) Upper bound 12.5 g

Lower bound 11.5 g **[2]**

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15 A doctor carried out a test on some one-year-old boys and girls.
He timed how long, in seconds, it took each of them to complete a task.

Freq density = Freq / width

This table shows the distribution of the times for the boys.

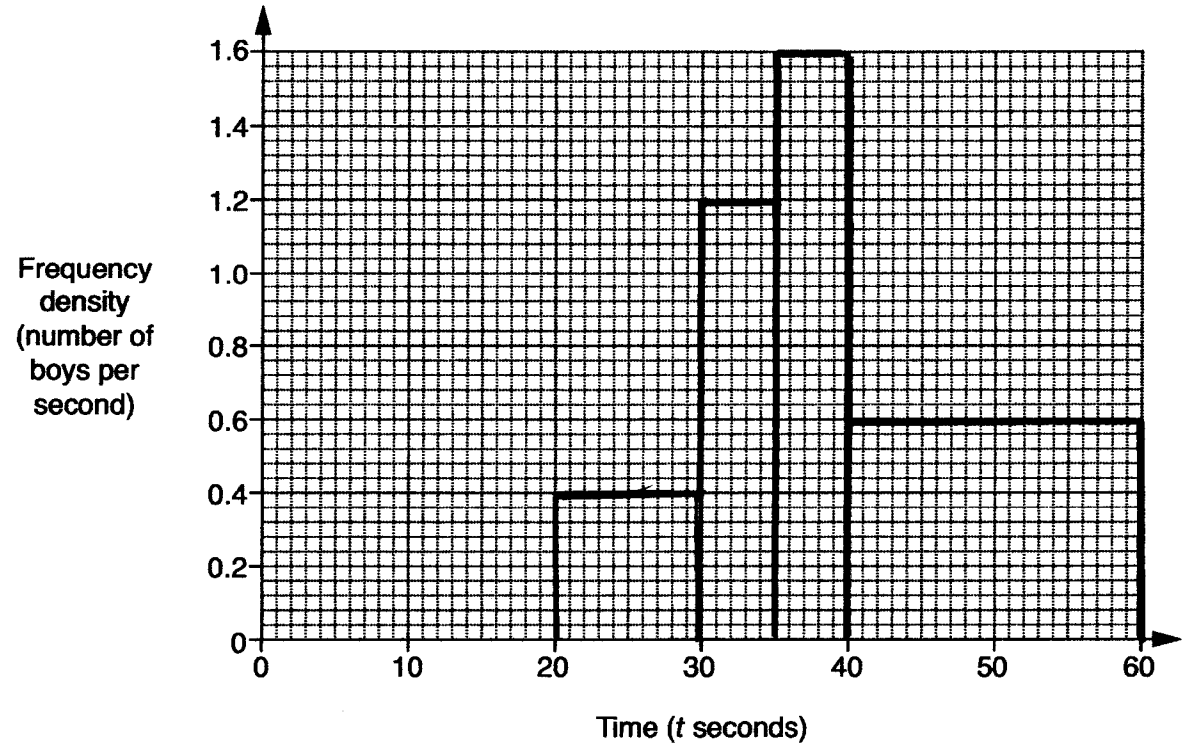
Time (t seconds)	Number of boys	Width	Freq Density
$20 \leqslant t < 30$	4	10	0·4
$30 \leqslant t < 35$	6	5	1·2
$35 \leqslant t < 40$	8	5	1·6
$40 \leqslant t < 60$	12	20	0·6

(a) Estimate the number of boys who took less than 25 seconds.

.....

(a) 2 **[1]**

(b) On the grid below, draw a histogram for the boys' times.



[3]

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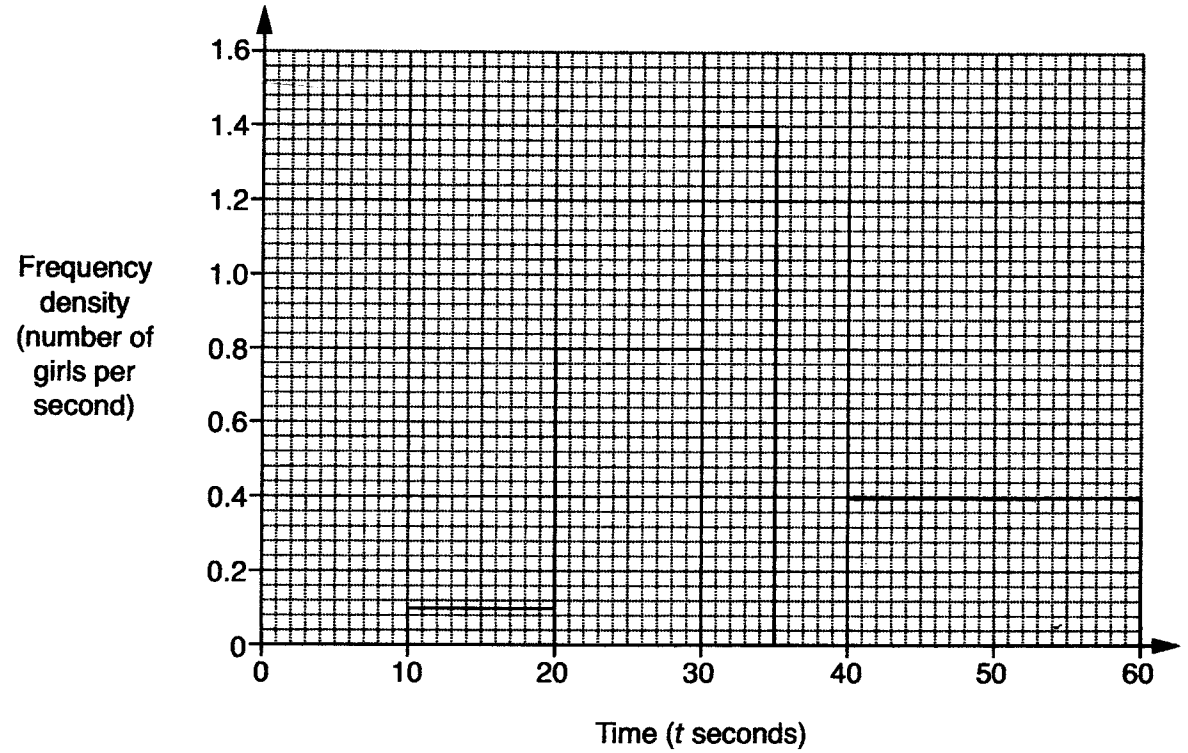
This histogram shows the distribution of the times for the girls.



(c) Estimate the number of girls who took more than 50 seconds.

$10 \times 0.4 = 4$

(c) 4 **[1]**

(d) Make one comparison between the distribution of the times of the boys and the girls.

On average girls take less time than boys

[1]

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16 A bus company wants to carry out a survey of students' views about its school bus service.

(a) One bus driver suggests asking every 10th student who gets on the bus.

Which word from this list best describes this type of sampling?

Random Systematic Stratified Quota

(a) Systematic **[1]**

(b) The company decides to ask a stratified sample of the 170 boys and 230 girls in Year 11.

Given that they will interview a sample of 80 of these students, how many boys and how many girls should there be in the sample?

$170 + 230 = 400$ students

Sampling fraction $= \frac{80}{400} = \frac{1}{5}$

$5 \overline{)170^{2}0}$ $5 \overline{)230^{3}0}$

34 46

(b) boys 34

girls 46 **[3]**

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17 **(a)** Write this expression as a single power of 2.

$\frac{2^{3x+2}}{2^{x+5}}$ $= 2^{(3x+2)-(x+5)}$

$= 2^{3x+2-x-5}$

$= 2^{2x-3}$

(a) 2^{2x-3} **[2]**

(b) You are given that $\frac{2^{3x+2}}{2^{x+5}} = 32$.

By writing 32 as a power of 2, find the value of x.

$2^{2x-3} = 2^5$

$2x - 3 = 5$

$2x = 5 + 3$

$2x = 8$

$x = \frac{8}{2}$

$x = 4$

(b) $x = 4$ **[3]**

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18 (a) Evaluate.

$64^{\frac{1}{2}} \times 2^{-4}$

Give your answer as simply as possible.

$= \sqrt{64} \times \frac{1}{2^4}$

$= 8 \times \frac{1}{16} = \frac{8}{16} = \frac{1}{2}$

(a) $\frac{1}{2}$ [3]

(b) Multiply out and simplify.

$(4 + 3\sqrt{7})(5 + 2\sqrt{7})$

$= 20 + 15\sqrt{7} + 8\sqrt{7} + 6 \times 7$

$= 62 + 23\sqrt{7}$

(b) $62 + 23\sqrt{7}$ [3]

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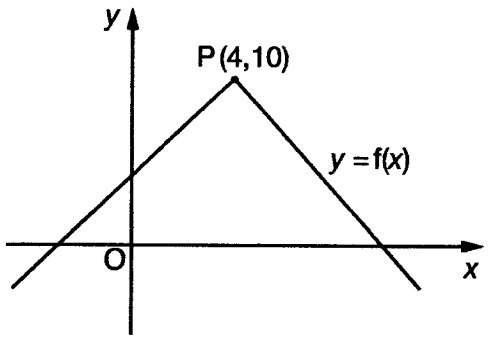
19 Here is the graph of $y = f(x)$.
The point P (4, 10) is a point on the graph.



NOT TO SCALE

What are the coordinates of the new position of P when the graph $y = f(x)$ is transformed to the graph of

(a) $y = 2f(x)$,

.....

(a) (4 , 20) **[1]**

(b) $y = f(x) - 3$?

.....

(b) (4 , 7) **[1]**

TURN OVER FOR QUESTION 20

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20 Solve algebraically these simultaneous equations.

$y = (x+5)(x-7) = x^2+5x-7x-35 = x^2-2x-35$

$y = 2x-3$

$y = x^2-2x-35$ (1)

$y = 2x-3$ (2)

Subst for y in (1)

$2x-3 = x^2-2x-35$

$0 = x^2-2x-35-2x+3$

$0 = x^2-4x-32$

$0 = (x+4)(x-8)$

$\Rightarrow \quad x+4=0 \quad$ or $\quad x-8=0$

$x = -4 \qquad x = 8$

when $x=-4$

$y = 2(-4)-3$

$= -8-3$

$= -11$

when $x=8$

$y = 2(8)-3$

$= 16-3$

$= 13$

$\begin{cases} x = -4 \\ y = -11 \end{cases} \qquad \begin{cases} x = 8 \\ y = 13 \end{cases}$

$x = \underline{-4}$ $y = \underline{-11}$

$x = \underline{8}$ $y = \underline{13}$ **[5]**

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