

A\* 83, A 66, B 49, C 33,



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GENERAL CERTIFICATE OF SECONDARY EDUCATION  
MATHEMATICS SYLLABUS A

J512/03

Paper 3 (Higher Tier)

MONDAY 19 MAY 2008

Morning  
Time: 2 hours

Candidates answer on the question paper  
Additional materials (enclosed): None

Solutions

Additional materials (required):  
Geometrical instruments  
Tracing paper (optional)



Candidate Forename

Candidate Surname

Centre Number

Candidate Number

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the boxes above.
- Use blue or black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Show your working. Marks may be given for a correct method even if the answer is incorrect.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Write your answer to each question in the space provided.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is **100**.



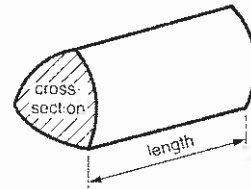
**WARNING**  
You are not allowed to use a  
calculator in this paper.

FOR EXAMINER'S USE

This document consists of 20 printed pages.

## Formulae Sheet: Higher Tier

**Volume of prism** = (area of cross-section) x length

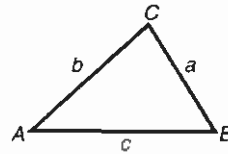


**In any triangle ABC**

**Sine rule**  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

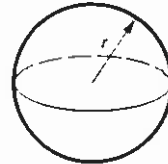
**Cosine rule**  $a^2 = b^2 + c^2 - 2bc \cos A$

**Area of triangle** =  $\frac{1}{2} ab \sin C$



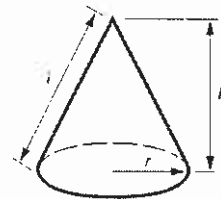
**Volume of sphere** =  $\frac{4}{3} \pi r^3$

**Surface area of sphere** =  $4\pi r^2$



**Volume of cone** =  $\frac{1}{3} \pi r^2 h$

**Curved surface area of cone** =  $\pi r l$



**The Quadratic Equation**

The solutions of  $ax^2 + bx + c = 0$ ,  
where  $a \neq 0$ , are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**PLEASE DO NOT WRITE ON THIS PAGE**

- 1 (a) A 100g box of breakfast cereal contains 20g of fruit and 80g of oats.

Write the ratio of fruit to oats in its simplest form.

$$\begin{aligned} & 20 : 80 \\ = & 1 : 4 \end{aligned}$$

(a) 1 : 4 [1]

- (b) What weight of fruit is in a 250g box of the same cereal?

$$1 + 4 = 5 \text{ parts}$$

$$1 \text{ part} = 250 \div 5 = 50\text{g}$$

(b) 50 g [2]

- (c) As a special offer, the 250g of cereal is increased by 20%.

What weight of cereal is in the special offer box?

$$10\% = 25\text{g}$$

$$20\% = 50\text{g}$$

$$250 + 50 = 300\text{g}$$

(c) 300 g [3]

- 2 (a) The probability that Rovers will win their next match is 0.6.  
The probability that Rovers will lose their next match is 0.15.

Work out the probability that Rovers will draw their next match.

$$\begin{aligned}
 P(\text{draw}) &= 1 - (0.6 + 0.15) \\
 &= 1 - 0.75 \\
 &= 0.25
 \end{aligned}$$

(a) 0.25 [2]

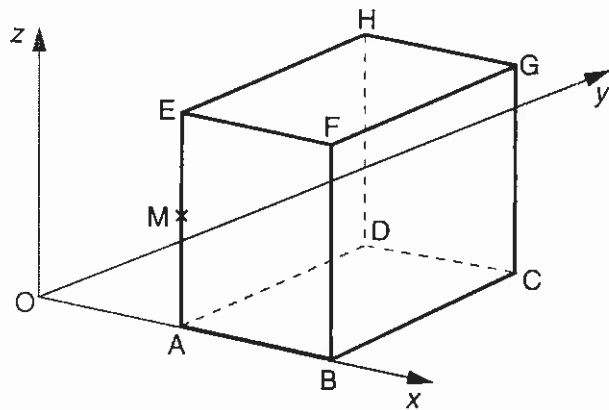
- (b) A weather man says 'There is a 40% chance of rain today.'  
Tracy says 'That means there is a 60% chance of it being sunny today.'

Explain why Tracy is **not** correct.

40% + 60% = 100%  
but there are other types of weather [1]  
eg cloudy, snowing

5

- 3 ABCDEFGH is a cuboid.  
With the  $x$ ,  $y$  and  $z$  axes shown,  
A is the point  $(3,0,0)$ ,  
B is the point  $(8,0,0)$ ,  
D is the point  $(3,6,0)$ ,  
E is the point  $(3,0,4)$ .



(a) Write down the coordinates of

(a)(i) C (8, 6, 0) [1]

(ii) F (8, 0, 4) [1]

(iii) G (8, 6, 4) [1]

(b) Work out the coordinates of M, the midpoint of AE.

$\left( \frac{3+3}{2}, \frac{0+0}{2}, \frac{0+4}{2} \right)$

(b) (3, 0, 2) [2]

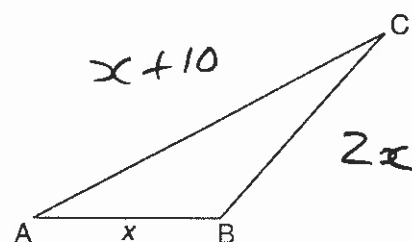
4 In triangle ABC,

- AB is  $x$  cm long
- BC is twice the length of AB
- AC is 10 cm longer than AB.

The perimeter of the triangle is 42 cm.

Write down an equation in  $x$  and solve it.

Use your answer to find the lengths of the sides of the triangle.



NOT TO SCALE

$$AB = x \quad BC = 2x \quad AC = x + 10$$

$$\text{Perimeter} = x + 2x + x + 10 = 42$$

$$4x + 10 = 42$$

$$4x = 42 - 10$$

$$4x = 32$$

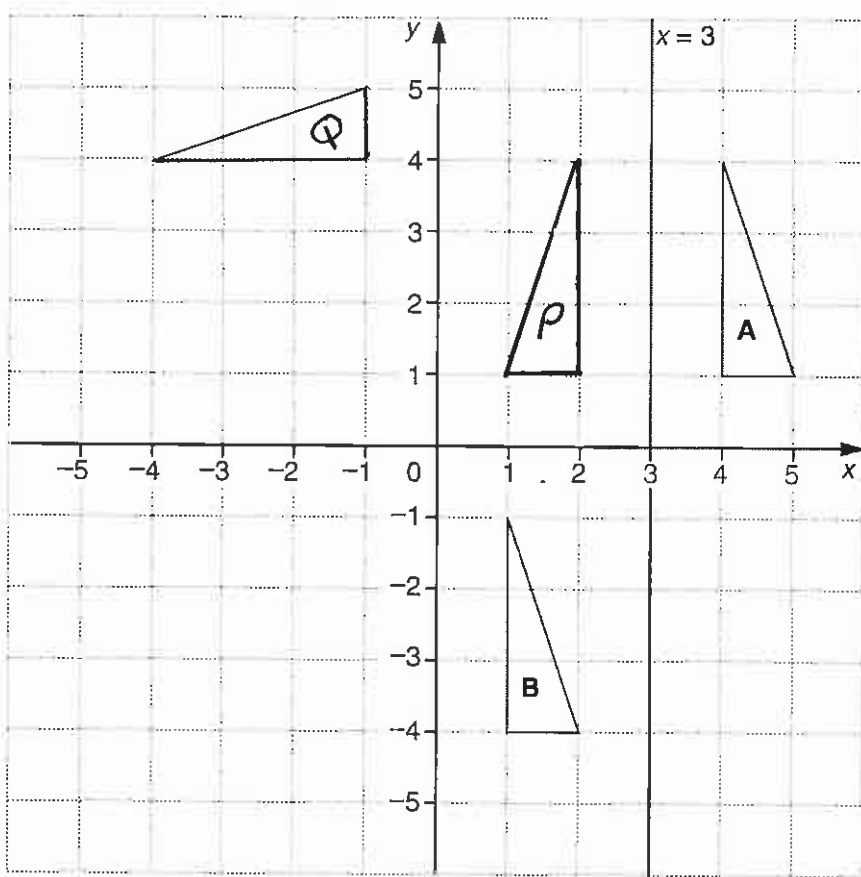
$$x = \frac{32}{4}$$

$$x = 8$$

$$\begin{aligned} x &= 8 \\ 2x &= 16 \\ x+10 &= 18 \end{aligned}$$

$$\underline{8} \text{ cm, } \underline{16} \text{ cm, } \underline{18} \text{ cm [5]}$$

5



- (a) Describe the translation that moves triangle **A** onto triangle **B**.

Translation by  $\begin{pmatrix} -3 \\ -5 \end{pmatrix}$

[1]

- (b) Reflect triangle **A** in the line  $x = 3$ .  
Label the image **P**.

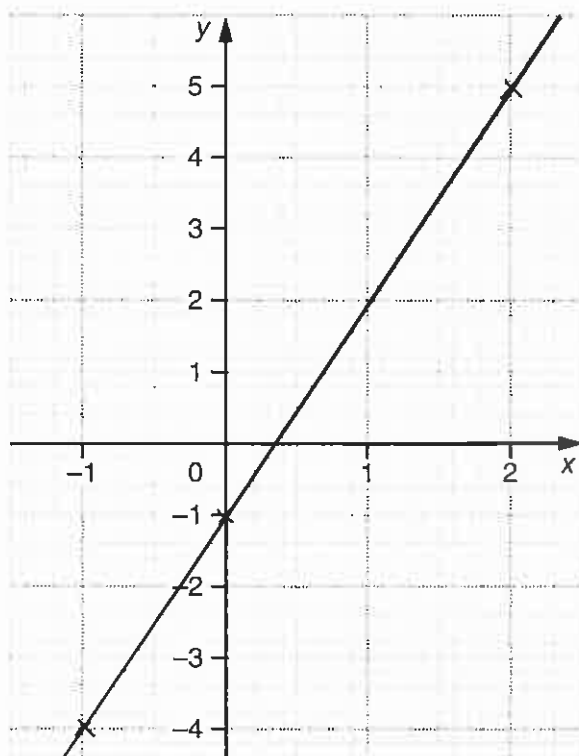
[1]

- (c) Rotate triangle **A**  $90^\circ$  anticlockwise about the origin.  
Label the image **Q**.

[3]

- 6 (a) (i) Draw the graph of  $y = 3x - 1$ .  
You may use the table below to help you.

$x$	-1	0	2
$y$	-4	-1	5



[3]

- (ii) Use your graph to solve the equation  $3x - 1 = 0$ .

(a)(ii)  $x = 0.3$  [1]

- (b) Rearrange  $y = 3x - 1$  to make  $x$  the subject.

$$y = 3x - 1$$

$$y + 1 = 3x$$

$$\frac{y + 1}{3} = x$$

(b)  $x = \frac{y + 1}{3}$  [2]



7 (a) Multiply out.

$$3(2x - 5) = 6x - 15$$

.....

.....

(a) 6x - 15 [1]

(b) Factorise completely.

$$4a + 2ac = 2a(2 + c)$$

.....

.....

(b) 2a(2 + c) [2]

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- 8 (a) Write 200 as a product of its prime factors, using powers.

$$\begin{array}{r}
 2 \overline{)200} \\
 \underline{2 \ 100} \\
 2 \overline{)50} \\
 \underline{5 \ 25} \\
 5 \overline{)25} \\
 \underline{5 \ 5} \\
 1
 \end{array}$$

$$\begin{aligned}
 200 &= 2 \times 2 \times 2 \times 5 \times 5 \\
 &= 2^3 \times 5^2
 \end{aligned}$$

(a)  $2^3 \times 5^2$  [3]

- (b) Simplify each of the following.

(i)  $y^5 \times y^3 = y^{5+3} = y^8$

(b)(i)  $y^8$  [1]

(ii)  $\frac{y^5}{y^3} = y^{5-3} = y^2$

(ii)  $y^2$  [1]

(iii)  $(2y^5)^3 = 2^3 \times y^{5 \times 3} = 8y^{15}$

(iii)  $8y^{15}$  [2]

- 9 (a) Here are the first four terms of a sequence.

$$\begin{array}{cccc} 7 & 12 & 17 & 22 \\ 5 & 10 & 15 & 20 \end{array}$$

Find the  $n^{\text{th}}$  term of this sequence.

Adding 5  $n^{\text{th}}$  term =  $5n + 2$

.....

.....

.....

(a)  $5n + 2$  [2]

- (b) (i) The  $n^{\text{th}}$  term of another sequence is given by

$$n^2 + 3n + 2.$$

Work out the first two terms of this sequence.

$$n = 1 \quad 1^2 + 3(1) + 2 = 1 + 3 + 2 = 6$$

.....

$$n = 2 \quad 2^2 + 3(2) + 2 = 4 + 6 + 2 = 12$$

.....

.....

(b)(i) 6 12 [2]

- (ii) Factorise.

$$n^2 + 3n + 2$$

$$+1 \quad +2$$

.....

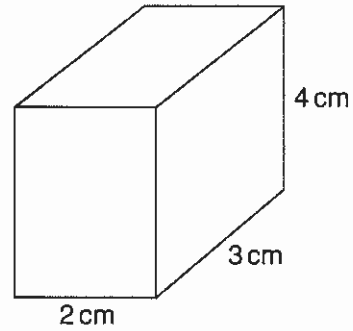
$$(n+1)(n+2)$$

.....

.....

(ii)  $(n+1)(n+2)$  [2]

- 10 The diagram shows a cuboid with dimensions 2 cm, 3 cm and 4 cm.



- (a) The mass of the cuboid is 36 g.

Work out the density of the cuboid.

$$\text{Volume} = 2 \times 3 \times 4 = 24 \text{ cm}^3$$

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} = \frac{36}{24} \text{ g/cm}^3$$

$$= 1.5 \quad (\text{a}) \quad \underline{1.5} \text{ g/cm}^3 \text{ [3]}$$

- (b) The mass of the cuboid is 36 g correct to the nearest gram.

Write down the greatest and least possible mass of the cuboid.

(b) greatest 36.5 g

least 35.5 g [2]

- 11 A theatre has 500 seats.  
The theatre manager recorded the audience size at each performance of a pantomime.  
The results are summarised in the table below.

Audience size ( $A$ )	$0 < A \leq 100$	$100 < A \leq 200$	$200 < A \leq 300$	$300 < A \leq 400$	$400 < A \leq 500$
Frequency	2	18	11	14	35

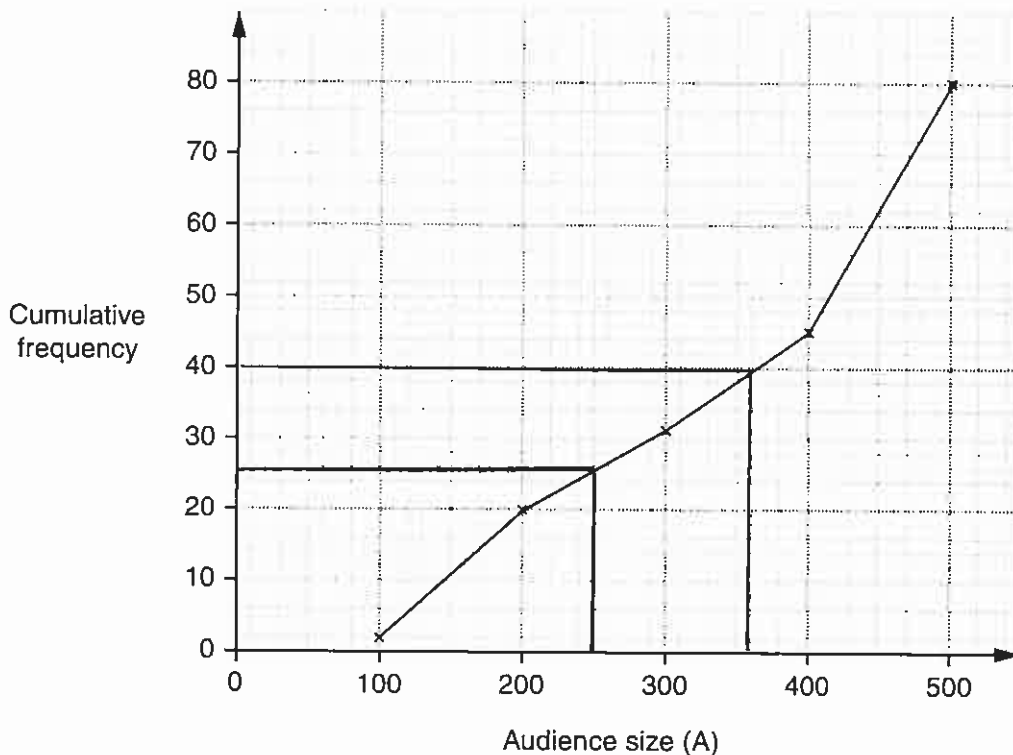
The cumulative frequency table for the audience size is given below.

Audience size ( $A$ )	$A \leq 100$	$A \leq 200$	$A \leq 300$	$A \leq 400$	$A \leq 500$
Cumulative frequency	2	20	31	45	80

- (a) Explain how the cumulative frequency values have been calculated.

A running total is kept, with each frequency being added to the previous total. [1]

- (b) The cumulative frequency graph for the audience size is shown below.



- (i) Use the graph to find an estimate of the median audience size.

(b)(i) 360 [1]

- (ii) Use the graph to estimate on how many occasions the theatre was **over** half full.

Under half full 26 times

(ii) 54 [2]

12 (a) (i) Write 845 000 in standard form.

$$8.45 \times 10^5$$

(a)(i)  $8.45 \times 10^5$  [1]

(ii) Write  $1.93 \times 10^{-4}$  as an ordinary number.

$$0.000193$$

(ii)  $0.000193$  [1]

(b) Change  $\frac{2}{9}$  into a recurring decimal.

$$\begin{array}{r} 0.222 \\ 9 \overline{) 2.000} \end{array} = 0.\dot{2}$$

(b)  $0.\dot{2}$  [2]

13 Solve algebraically.

$$\frac{7x-3}{2} = 2x+9$$

$$7x - 3 = 2(2x + 9)$$

$$7x - 3 = 4x + 18$$

$$7x - 4x = 18 + 3$$

$$3x = 21$$

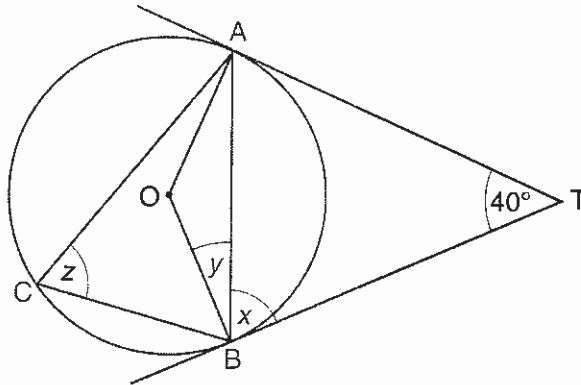
$$x = \frac{21}{3}$$

$$x = 7$$

$x = 7$  [4]

- 14 A, B and C are points on a circle, centre O.  
TA and TB are tangents to the circle.  
Angle ATB =  $40^\circ$ .

NOT TO  
SCALE



- (a) Find angle  $x$ .  
Give a reason for your answer.

$$x = \frac{180 - 40}{2} = \frac{140}{2} = 70^\circ$$

$x =$  70° because Tangents equal length so isosceles  $\Delta$  [2]

- (b) Find angle  $y$ .  
Give a reason for your answer.

$$90 - 70 = 20^\circ$$

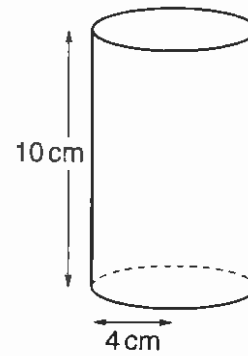
$y =$  20° because Angle between radius and tangent =  $90^\circ$  [2]

- (c) Find angle  $z$ .

$$z = 70^\circ \text{ (Alternate segment theorem)}$$

(c)  $z =$  70° [2]

- 15 (a) A cylinder has base radius 4 cm and height 10 cm.



Work out the volume of the cylinder.  
Leave your answer as a multiple of  $\pi$ .  
Give the units of your answer.

$$\begin{aligned} \text{Volume} &= \pi r^2 h \\ &= \pi \times 4^2 \times 10 \\ &= \pi \times 16 \times 10 \\ &= 160\pi \text{ cm}^3 \end{aligned}$$

(a) 160 $\pi$  cm<sup>3</sup> [3]

- (b) Two cones are mathematically similar.  
The height of the smaller cone is exactly half the height of the larger cone.  
The surface area of the **larger cone** is  $112\pi \text{ cm}^2$ .

Work out the surface area of the smaller cone.  
Leave your answer as a multiple of  $\pi$ .

$$\begin{aligned} \text{Length} & 1 : 2 \\ \text{Area} & 1 : 2^2 = 1 : 4 \end{aligned}$$

$$\text{Surface area of small cone} = \frac{112\pi}{4}$$

(b) 28 $\pi$  cm<sup>2</sup> [3]



16 You are given that  $y$  is inversely proportional to  $x$ , and that  $y = 9$  when  $x = 4$ .

(a) (i) Find an equation connecting  $y$  and  $x$ .

$$y = \frac{k}{x} \quad \text{Subst } y=9, x=4$$

$$9 = \frac{k}{4} \quad \Rightarrow \quad 9 \times 4 = k$$

$$36 = k \quad \text{so } y = \frac{36}{x}$$

$$(a)(i) \quad y = \frac{36}{x} \quad [3]$$

(ii) Find  $y$  when  $x = \frac{1}{2}$ .

$$\text{When } x = \frac{1}{2} \quad y = \frac{36}{\frac{1}{2}} = 36 \times 2 = 72$$

$$(ii) \quad y = 72 \quad [1]$$

(b) Find  $y$  when  $x = y$ .

$$y = \frac{36}{x} \quad \text{When } x = y \quad y = \frac{36}{y}$$

$$\Rightarrow y^2 = 36$$

$$\Rightarrow y = \sqrt{36} = \pm 6$$

$$(b) \quad y = \pm 6 \quad [2]$$

- 17 (a) There are three species of bat in a large cave.

A researcher wishes to select a representative sample of 40 of these bats to see what diseases they carry.

A count of the number of bats has given the following data.

Species of bat	A	B	C	Total
Number of bats	980	1290	1730	4000

Work out how many of each species should be selected for the sample.

$$\text{Sampling Fraction} = \frac{40}{4000} = \frac{1}{100}$$

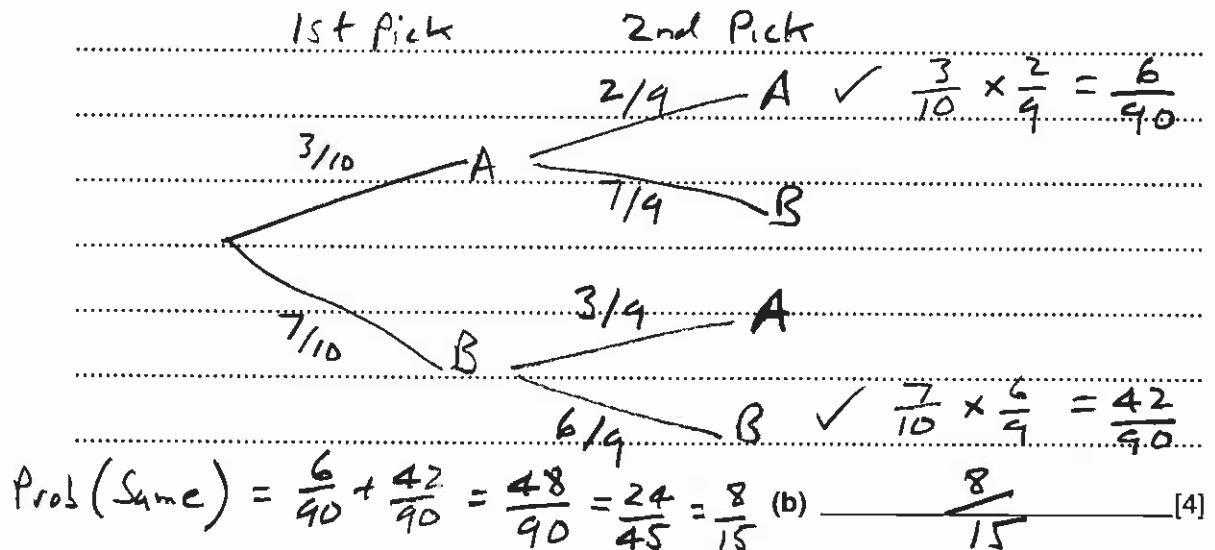
Choose 9.8 A 12.9 B 17.3 C

Round to 10 A 13 B 17 C

(a) A 10 B 13 C 17 [3]

- (b) In a small cave there are only ten bats, 3 of species A and 7 of species B. Two of these bats are randomly selected, at the same time.

Work out the probability that both are the same species.

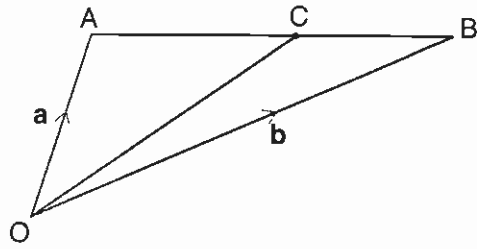


OK to leave as  $\frac{48}{90}$

- 18 In the triangle,  $\vec{OA} = \mathbf{a}$ ,  $\vec{OB} = \mathbf{b}$   
and  $\vec{AC} = t\vec{AB}$ .

$$\vec{OC} = p\mathbf{a} + q\mathbf{b}.$$

Find  $p$  and  $q$  in terms of  $t$ .



$$\vec{AB} = \underline{\underline{\mathbf{b} - \mathbf{a}}}$$

$$\vec{OC} = \vec{OA} + \vec{AC} = \vec{OA} + t\vec{AB}$$

$$= \underline{\underline{\mathbf{a}}} + t(\underline{\underline{\mathbf{b} - \mathbf{a}}})$$

$$= \underline{\underline{\mathbf{a}}} + t\underline{\underline{\mathbf{b}}} - t\underline{\underline{\mathbf{a}}}$$

$$= (1-t)\underline{\underline{\mathbf{a}}} + t\underline{\underline{\mathbf{b}}}$$

$$p = \underline{1-t} \quad q = \underline{t} \quad [4]$$

- 19 (a) (i) Simplify.

$$\sqrt{20} \times \sqrt{5} = \sqrt{20 \times 5}$$

$$= \sqrt{100}$$

$$= 10$$

$$(a)(i) \quad \underline{10} \quad [2]$$

- (ii) Rationalise the denominator and simplify.

$$\frac{20}{\sqrt{5}}$$

$$\frac{20}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{20\sqrt{5}}{5} = 4\sqrt{5}$$

$$(ii) \quad \underline{4\sqrt{5}} \quad [2]$$

- (b) Change  $0.\dot{4}0\dot{3}$  to a fraction.

$$\text{Let } x = 0.\dot{4}0\dot{3} = 0.403403403 \quad \textcircled{1}$$

$$1000x = 403.403403 \quad \textcircled{2}$$

Subtract  
② - ①

$$999x = 403$$

$$x = \frac{403}{999}$$

$$(b) \quad \underline{\frac{403}{999}} \quad [2]$$

- 20 Josie is buying a snooker cue and a carrying case.  
The cue that she likes best is 142.6 cm long, correct to the nearest millimetre.  
The carrying case that she likes best has internal length of 140 cm, correct to the nearest 5 cm.

Is it possible that the cue will fit into the carrying case?  
Show your reasoning clearly.

142.6 cm to nearest mm

---

$\Rightarrow 142.55 \text{ cm} \leq \text{cue} \leq 142.65 \text{ cm}$

---

case 140 cm to nearest 5 cm

---

$\Rightarrow 137.5 \leq \text{case} \leq 142.5$  [2]

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No not possible because  
minimum length of cue = 142.55 cm  
and max length of case is only 142.5 cm

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