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Solutions

H

GENERAL CERTIFICATE OF SECONDARY EDUCATION

MATHEMATICS SYLLABUS A
Paper 4 (Higher Tier)

J512/04
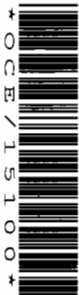
Candidate Forename		Candidate Surname	

| Centre Number						Candidate Number				

INSTRUCTIONS TO CANDIDATES

• Write your name clearly in capital letters, your Centre Number and Candidate Number in the boxes above.
• Use black ink. Pencil may be used for graphs and diagrams only.
• Read each question carefully and make sure that you know what you have to do before starting your answer.
• Show your working. Marks may be given for a correct method even if the answer is incorrect.
• Answer **all** the questions.
• Do **not** write in the bar codes.
• Write your answer to each question in the space provided, however additional paper may be used if necessary.

INFORMATION FOR CANDIDATES

• The number of marks is given in brackets [ ] at the end of each question or part question.
• You are expected to use an electronic calculator for this paper.
• Use the π button on your calculator or take π to be 3.142 unless the question says otherwise.
• The total number of marks for this paper is **100**.
• This document consists of **20** pages. Any blank pages are indicated.

Formulae Sheet: Higher Tier

Area of trapezium $= \frac{1}{2}(a + b)h$
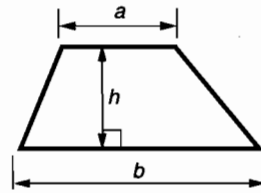
Volume of prism = (area of cross-section) × length
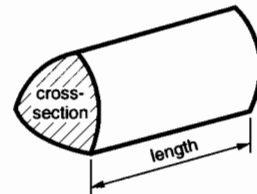
In any triangle ABC

Sine rule $\quad \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle $= \frac{1}{2}ab \sin C$
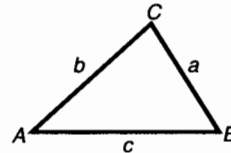
Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$
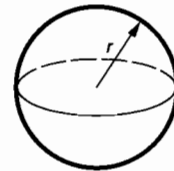
Volume of cone $= \frac{1}{3}\pi r^2 h$

Curved surface area of cone $= \pi r l$

The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$,
where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

PLEASE DO NOT WRITE ON THIS PAGE

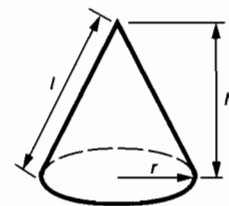1   Calculate.

(a) $\frac{7.8 - 3.1}{1.2 + 6.9}$ = $\frac{(7.8 - 3.1)}{(1.2 + 6.9)}$ = 0.5802469136

(a) ___0.580___ to 3 s.f. [2]

(b) $\sqrt{2.56^2 - 1.4^2}$

= $\sqrt{(2.56^2 - 1.4^2)}$ = 2.143268532

(b) ___2.14___ to 3 s.f. [2]

2   Jayne uses these ingredients to make play dough.

Play dough for 2 children	
Plain flour	225 g
Oil	2 tablespoons
Water	¾ pint
Salt	140 g

10 children

= 5 × 2 children

(a) Jayne wants to make enough play dough for 10 children.

Work out the amount of water Jayne will need.

Water $\frac{3}{4}$ × 5

= $\frac{15}{4}$

(a) = $3\frac{3}{4}$ _____ pints [2]

(b) Jayne has lots of oil and water, but only a 1.5 kg bag of plain flour and a 1 kg bag of salt.

What is the maximum number of children Jayne can make play dough for?
You must show your working.

Salt for 1 child = 140g ÷ 2 = 70g

1000 ÷ 70 = 14.28 so salt limited to 14 children

Plain flour for 1 child = 225 ÷ 2 = 112.5g

1500 ÷ 112.5 = 13.33 so flour limited to 13 children

Take lower limit
of these two answers

(b) ___13 children___ [3]

3 **(a)** One question on the 2001 Census form was:

'How many cars are available for use by one or more members of your household?'

There was space on the form to write down who lived at that household.
Jenna collects information about the number of people and the number of cars at each household from a sample of 100 Census forms.

In this sample there were no households where more than 5 people lived and none had more than 3 cars.

(i) Design a two-way table for Jenna to use. [3]

Number of People

	1	2	3	4	5	Total
0						
1						
2			14			
3						
Total						

Number of Cars

(ii) In Jenna's sample there are 14 households with 3 people and 2 cars.

Show this data in your table in part **(a)(i)**. [1]

(b) Jenna uses this question in a survey.

'How many bicycles are there in your household?'

None ☐   1 – 2 ☐   More than 3 ☐

What mistake has Jenna made?

No box suitable for 3 bicycles

_____ [1]

4 **(a)** A parallelogram has angles as shown.
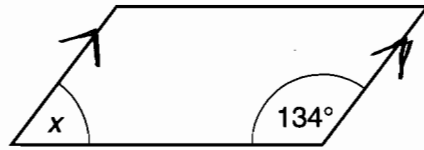
NOT TO
SCALE

Work out angle x.
Give a reason for your answer.

$180 - 134 = 46$

x = _____46_____ ° because ___allied angles add up___
___to 180°___ **[2]**

(b) A quadrilateral has angles as shown.
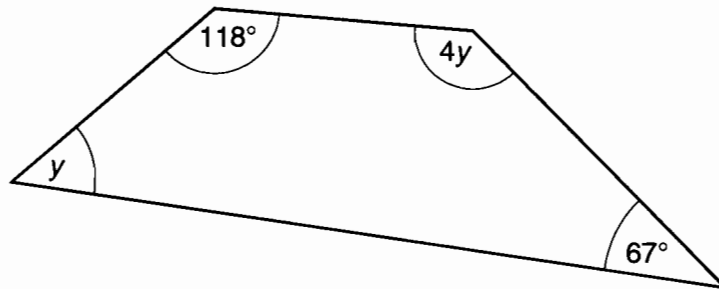
NOT TO
SCALE

Angles of any quadrilateral add up to 360°

Work out angle y.

$y + 118 + 4y + 67 = 360$

$5y = 360 - 118 - 67$

$5y = 175$

$y = \frac{175}{5} = 35$

(b) _____$y = 35$_____ ° **[4]**

Turn over

5 Gary's dogs eat 6 tins of dog food between them each day.
The tins are sold in boxes of 44.
Gary normally buys one box of 44 tins for each week.

Explain, showing your calculations, why Gary does not have to buy a box for the 22nd week.

6 tins per day $\Rightarrow$ 6 × 7 = 42 tins per week

For 22 weeks he requires 42 × 22 = 924 tins

21 × 44 = 924 so 21 boxes will provide [3]

enough for 22 weeks

6 **(a)** The nth term of a sequence is given by $n^2 - 2$.

Work out the first three terms of this sequence.

$1^2 - 2 = 1 - 2 = -1$
$2^2 - 2 = 4 - 2 = 2$
$3^2 - 2 = 9 - 2 = 7$

(a) $\underline{-1}$ $\underline{2}$ $\underline{7}$ [2]

(b) In another sequence of three numbers, the difference between each number and the next is 4.
The total of the three numbers is 6.

What are the three numbers?

Let numbers be

a, $a + 4$, $a + 8$

$\Rightarrow$ $a + a + 4 + a + 8 = 6$

$3a + 12 = 6$

$3a = 6 - 12$

$3a = -6$ (b) $\underline{-2}$ $\underline{2}$ $\underline{6}$ [2]

$a = \dfrac{-6}{3}$

$a = -2$

So numbers are -2

$-2 + 4 = 2$

$-2 + 8 = 6$

7
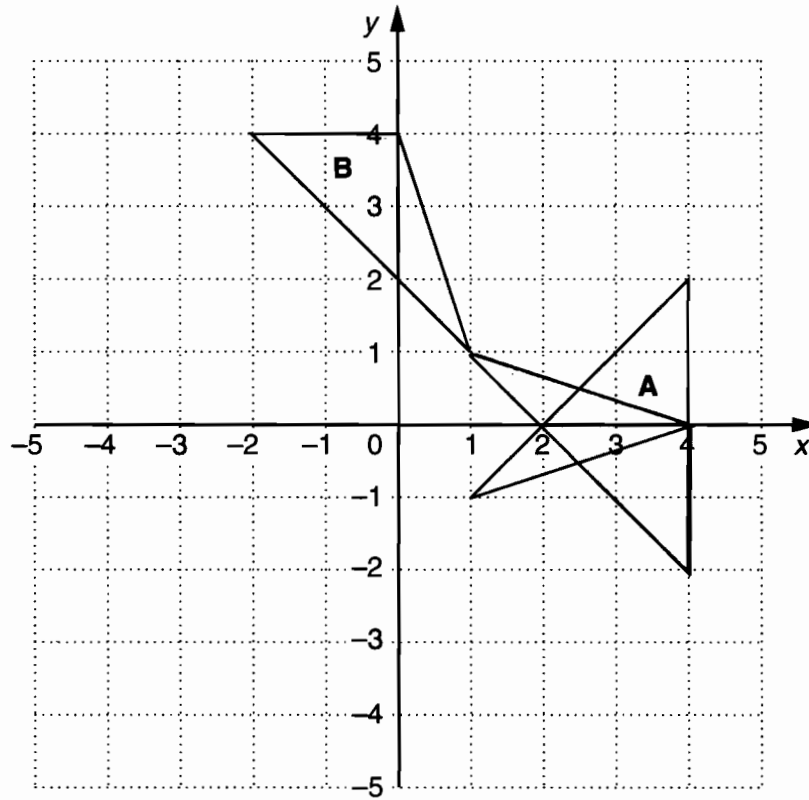
(a) Describe fully the **single** transformation that maps triangle **A** onto triangle **B**.

Rotation by 90° anti-clockwise about (0,0)

_____ [3]

(b) Draw the reflection of triangle **A** in the x-axis. [2]

Turn over

8   Tom made an electronic dice which gave scores of 1, 2, 3, 4, 5 or 6.
     This table summarises 100 scores.

Score	Frequency
1	17
2	19
3	15
4	17
5	18
6	14

100

Freq x Score

17 x 1 = 17
19 x 2 = 38
15 x 3 = 45
17 x 4 = 68
18 x 5 = 90
14 x 6 = 84
         342

(a) Work out the mean score.

$$\frac{342}{100} = 3.42$$

(a) _____3.42_____ [3]

(b) Is the dice biased?
     Give a reason for your answer.

Dice does not appear to be biased
because the frequencies for each score [1]
are similar.

9 Simplify.

(a) $d^7 \times d^4$ $= d^{7+4} = d^{11}$

(a) _____ d^{11} _____ [1]

(b) $\dfrac{d^9}{d^3}$ $= d^{9-3} = d^6$

(b) _____ d^6 _____ [1]

10 A cylindrical tin has radius 4.2 cm and height 9.1 cm.

4.2 cm

9.1 cm

Circumference of circle = $2\pi r$

Work out the curved surface area of the tin.
Give your answer to an appropriate degree of accuracy.

Area of curved surface = $2\pi r \times$ height

= $2 \times \pi \times 4.2 \times 9.1$

= 240.14

_____ 240 _____ cm^2 [4]

Turn over

11 **(a)** Work out the integer values of *n* that satisfy this inequality.

$$7 < 4n \leqslant 20 \qquad \frac{7}{4} < n \leqslant \frac{20}{4}$$

....

$$1.75 < n \leqslant 5$$

....

(a) _2, 3, 4, 5_ **[3]**

(b) On the grid, indicate clearly the region that satisfies all these inequalities.

$$x \geqslant 2 \qquad\qquad y \geqslant 3 \qquad\qquad x + y \leqslant 7$$

[3]

12

NOT TO SCALE

14.4 cm

5.7 cm

12.9 cm
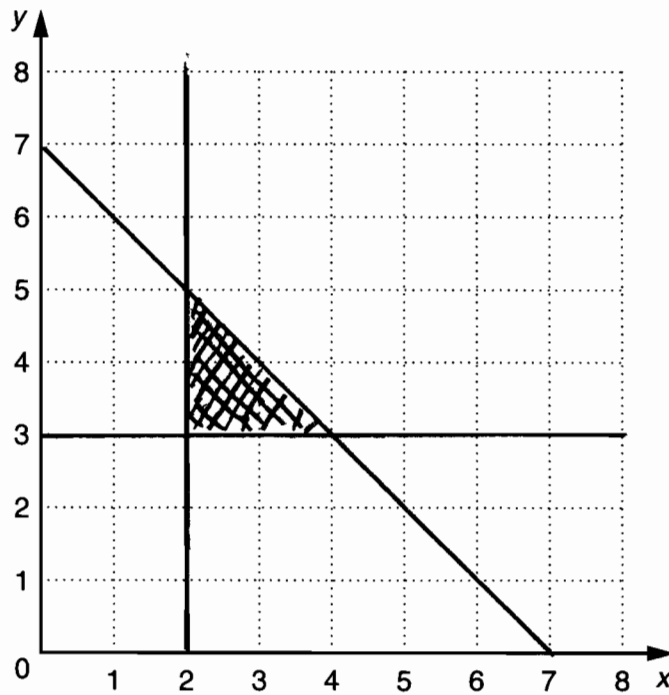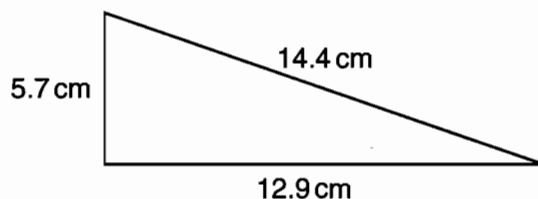
Is a triangle with these lengths right-angled?
Explain your answer using calculations.

If right-angled Pythagoras theorem applies

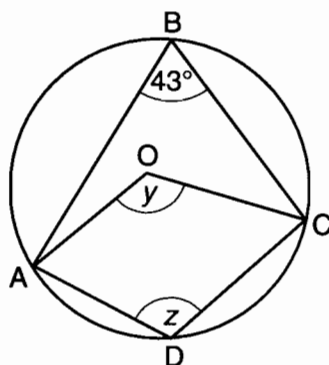Should have $5.7^2 + 12.9^2 = 14.4^2$

$198.9 = 207.36$ ✗

Not true so not right-angled [3]

13 The points A, B, C and D lie on the circumference of a circle, centre O.

NOT TO SCALE

Find the size of angle y and angle z.
Give a reason for each answer.

....

....

$y =$ __86__ ° because angle at centre is twice angle at circumference

$z =$ __137__ ° because opposite angles of a cyclic quad add up to 180° [4]

Turn over

14 Rectangle ABCD is made from five **squares**.

B _____ C
5x
3x
3x | 5x   NOT TO SCALE
x
2x
x
A   D
x
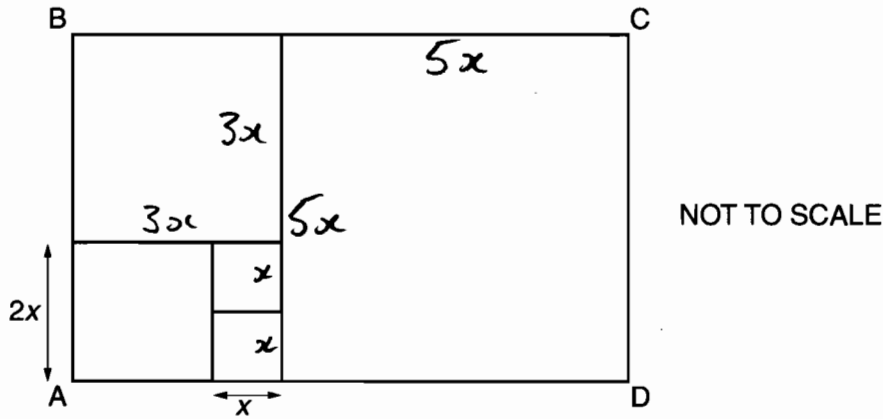
The area of rectangle ABCD is 810cm^2.

Work out the value of x.
Show all your working.

$$\text{Total Area} = x^2 + x^2 + (2x)^2 + (3x)^2 + (5x)^2$$

$$= x^2 + x^2 + 4x^2 + 9x^2 + 25x^2$$

$$= 40x^2$$
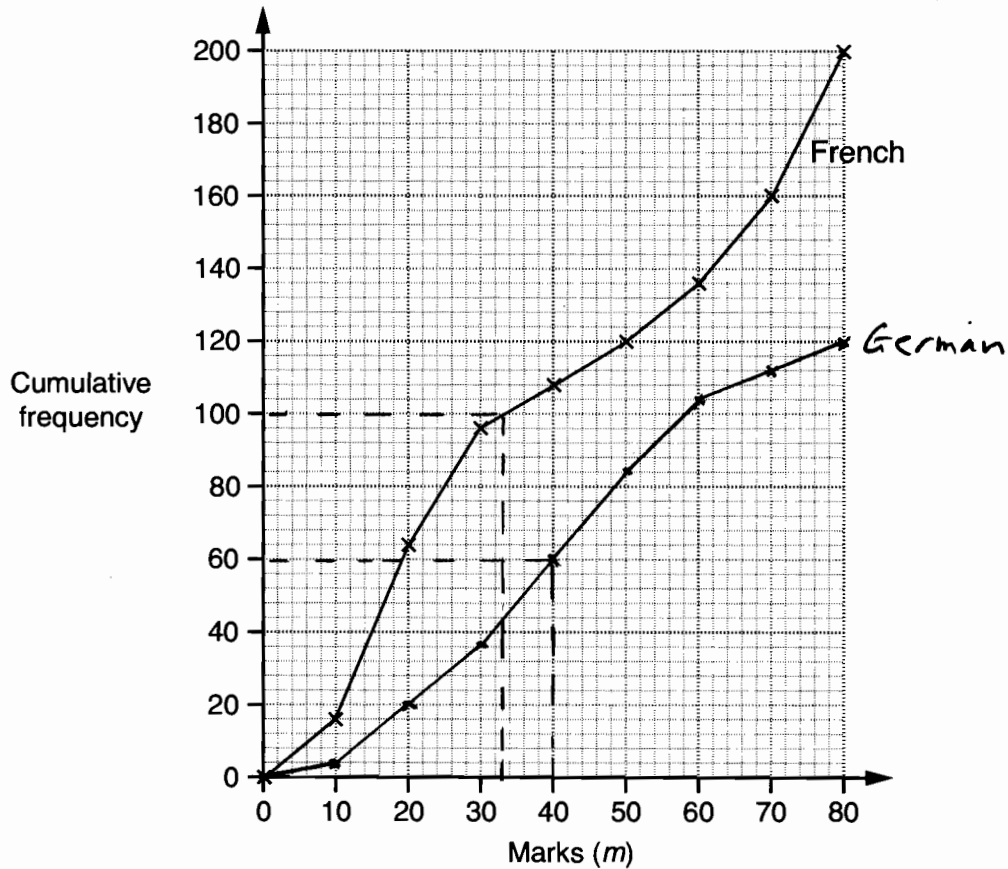
$$40x^2 = 810$$

$$x^2 = \frac{810}{40} = \frac{81}{4} \implies x \sqrt{\frac{81}{4}} = \frac{\sqrt{81}}{\sqrt{4}} = \frac{9}{2}$$

_____ cm **[5]**

$$x = 4\tfrac{1}{2} \text{ cm}$$

15 The cumulative frequency graph summarises the marks obtained in a French exam by 200 students.

The table gives the cumulative frequencies of marks obtained in a German exam by 120 students.

Marks (m)	$m \leqslant 10$	$m \leqslant 20$	$m \leqslant 30$	$m \leqslant 40$	$m \leqslant 50$	$m \leqslant 60$	$m \leqslant 70$	$m \leqslant 80$
Cumulative frequency	4	20	36	60	84	104	112	120

(a) On the grid, draw a cumulative frequency graph to summarise the marks obtained by the students in the German exam. **[3]**

(b) In which exam, French or German, was the median mark higher, and by how much?

French median = 33

German median = 40

German higher by 7

(b) German by 7 marks **[2]**

16 (a) Factorise.

(i) $x^2 - 8x$

$x(x - 8)$

(a)(i) $\underline{x(x-8)}$ **[1]**

(ii) $6x^3 + 10xy^3$

$2x(3x^2 + 5y^3)$

(ii) $\underline{2x(3x^2 + 5y^3)}$ **[2]**

(iii) $4x^2 - y^2$ $= (2x)^2 - y^2 = (2x+y)(2x-y)$

(iii) $\underline{(2x+y)(2x-y)}$ **[2]**

(b) Simplify.

$$\frac{x^2 + 3x}{3x^2} = \frac{x(x+3)}{3x^2} = \frac{x+3}{3x}$$

(b) $\underline{\dfrac{x+3}{3x}}$ **[2]**

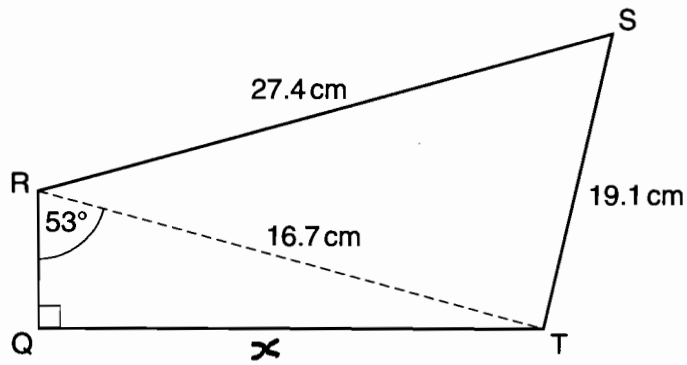**17 (a)** Write 91 000 000 in standard form.

(a) $\underline{9.1 \times 10^7}$ **[1]**

(b) A picometre is 10^{-12} m.
A nanometre is 10^{-9} m.

$10^{-9} \div 10^{-12} = 10^{-9--12} = 10^{-9+12}$

How many picometres are there in a nanometre?

$= 10^3$

$= 1000$

(b) $\underline{1000}$ **[2]**

18

27.4 cm

S

NOT TO
SCALE

R

53°

16.7 cm

19.1 cm

Q

x

T

$$Sin = \frac{O}{H}$$

(a) Calculate length QT.

$$Sin\,53° = \frac{x}{16.7}$$

$$x = 16.7 \times sin\,53° \qquad = 13.337$$

(a) $\underline{\quad 13.3 \quad}$ cm [3]

(b) Calculate angle RST. Cosine Rule $\quad cos\,S = \dfrac{r^2 + t^2 - s^2}{2rt}$

$$cos\,S = \frac{\left(19.1^2 + 27.4^2 - 16.7^2\right)}{\left(2 \times 19.1 \times 27.4\right)} = 0.7993656132$$

$$S = cos^{-1}\left(0.7993656132\right) = 36.9°$$

(b) $\underline{\quad 36.9 \quad}$ ° [3]

Turn over

19 The time series graph shows the amounts spent by a school on exams in January and in June each year.
The two-point moving averages (•) are also shown.
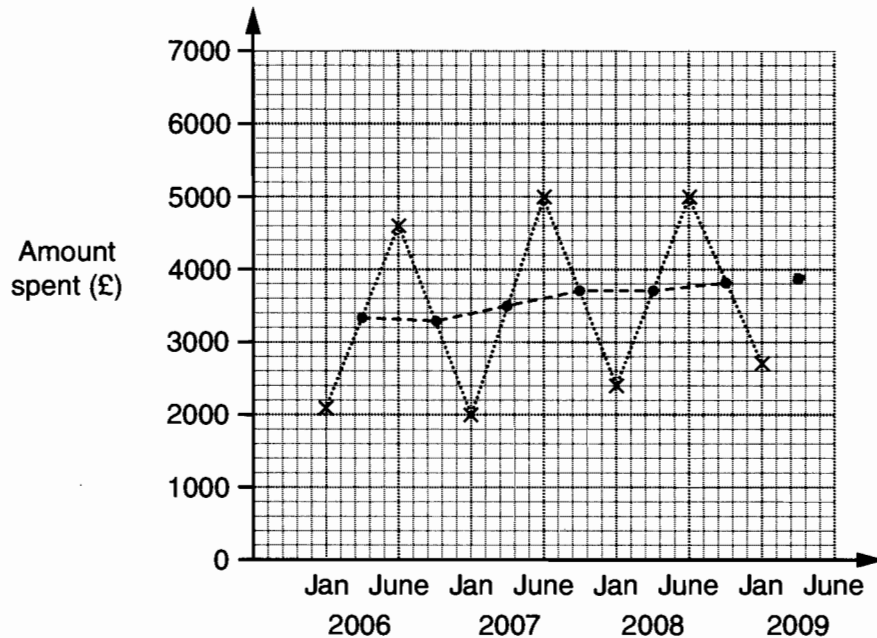
(a) Give a reason why it is appropriate to use a two-point moving average.

Exams are twice per year

_____ **[1]**

(b) Predict the next moving average and use this to work out an estimate of the amount spent on exams by the school in June 2009.

From graph predicted moving average for 2009
= £3900
so   Jan09 cost + June09 cost = 2 × £3900 = £7800
      June 09 cost = £7800 − Jan09 cost = £7800 − £2700

(b) £ _____5,100_____ **[3]**

20 Use trial and improvement to solve $7^x = 27$.
Give your answer correct to 2 decimal places.
Show all your trials and their outcomes.

x	7^x	
2	7^2	$= 49$
1.5	$7^{1.5}$	$= 18.52$
1.7	$7^{1.7}$	$= 27.33$
1.65	$7^{1.65}$	$= 24.80$
1.68	$7^{1.68}$	$= 26.29$
1.64	$7^{1.64}$	$= 26.80$
1.695	$7^{1.695}$	$= 27.06$ too big so round down to 1.69

$x = 1.69$ _____ **[4]**

21 (a) Express $\sqrt{45}$ in the form $a\sqrt{b}$, where a and b are integers and b is as small as possible.

$\sqrt{45} = \sqrt{9 \times 5} = 3\sqrt{5}$

(a) _____ $3\sqrt{5}$ _____ **[1]**

(b) Rationalise the denominator of $\dfrac{9}{\sqrt{6}}$.
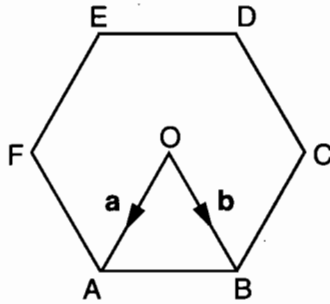
Give your answer in its simplest form.

$\dfrac{9}{\sqrt{6}} = \dfrac{9}{\sqrt{6}} \times \dfrac{\sqrt{6}}{\sqrt{6}} = \dfrac{9\sqrt{6}}{6}$

$= \dfrac{3\sqrt{6}}{2}$

(b) _____ $\dfrac{3\sqrt{6}}{2}$ _____ **[2]**

Turn over

22 ABCDEF is a regular hexagon, centre O.

$$\overrightarrow{OA} = \mathbf{a}. \qquad \overrightarrow{OB} = \mathbf{b}.$$

(a) Find in terms of **a** and **b** the vectors

(i) $\overrightarrow{CB}$, $\qquad \overrightarrow{CB} = \overrightarrow{OA} = \underline{a}$

(a)(i) _____ $\underline{a}$ _____ **[1]**

(ii) $\overrightarrow{DB}$. $\qquad \overrightarrow{DB} = \overrightarrow{DC} + \overrightarrow{CB}$
$\qquad\qquad\quad = \underline{b} + \underline{a}$

(ii) _____ $\underline{b} + \underline{a}$ _____ **[1]**

(b) X lies on DB such that DX : XB = 1 : 2.

Find $\overrightarrow{OX}$, in terms of **a** and **b**.
Give your answer in a simplified form.

$$\overrightarrow{OX} = \overrightarrow{OD} + \overrightarrow{DX}$$
$$= \overrightarrow{OD} + \tfrac{1}{3}\overrightarrow{DB}$$
$$= -\underline{a} + \tfrac{1}{3}(\underline{b} + \underline{a})$$
$$= -\underline{a} + \tfrac{1}{3}\underline{b} + \tfrac{1}{3}\underline{a}$$

(b) _____ $-\tfrac{2}{3}\underline{a} + \tfrac{1}{3}\underline{b}$ _____ **[3]**

23 **(a)** Explain why $(x - y)^2 \geqslant 0$.

Positive² is positive

Negative² is positive

Zero² is 0 $\therefore (x - y)^2 \geqslant 0$

[2]

(b) Hence, show that $x^2 + y^2 \geqslant 2xy$.

[1]

$$(x - y)^2 = (x - y)(x - y)$$

$$= x^2 - xy - xy + y^2$$

$$= x^2 + y^2 - 2xy$$

From part (a) $(x - y)^2 \geqslant 0$

$$\therefore \quad x^2 + y^2 - 2xy \geqslant 0$$

$$x^2 + y^2 \geqslant 2xy$$