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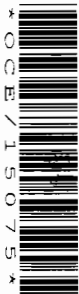
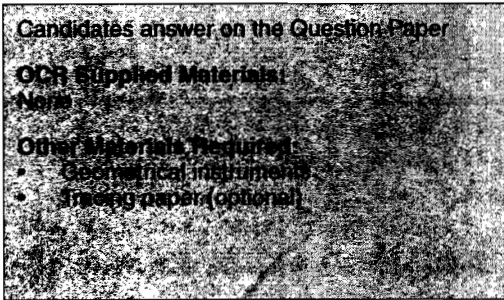
GENERAL CERTIFICATE OF SECONDARY EDUCATION
MATHEMATICS SYLLABUS A
Paper 3 (Higher Tier)

J512/03

Solutions

Tuesday 12 January 2010
Morning

Duration: 2 hours



Candidate Forename		Candidate Surname	
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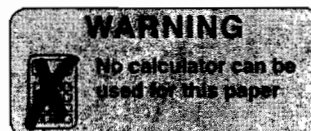
Centre Number						Candidate Number				
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INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the boxes above.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Show your working. Marks may be given for a correct method even if the answer is incorrect.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Write your answer to each question in the space provided, however additional paper may be used if necessary.

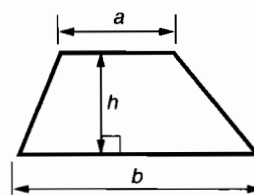
INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- The total number of marks for this paper is **100**.
- This document consists of **24** pages. Any blank pages are indicated.

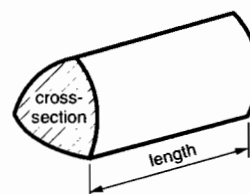


Formulae Sheet: Higher Tier

Area of trapezium $= \frac{1}{2}(a + b)h$



Volume of prism $= (\text{area of cross-section}) \times \text{length}$

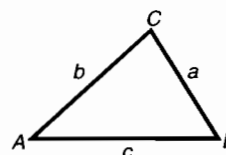


In any triangle ABC

Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

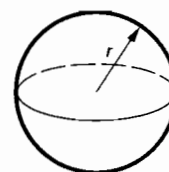
Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle $= \frac{1}{2} ab \sin C$



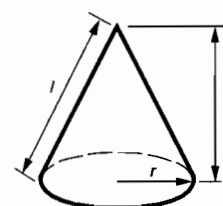
Volume of sphere $= \frac{4}{3} \pi r^3$

Surface area of sphere $= 4\pi r^2$



Volume of cone $= \frac{1}{3} \pi r^2 h$

Curved surface area of cone $= \pi r l$



The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$,
where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

PLEASE DO NOT WRITE ON THIS PAGE

- 1 There are blue, red, green and yellow counters in a box.
A counter is taken at random from the box.

(a) Complete the table.

$$0.05 + 0.15 + 0.35 = 0.55$$

$$1 - 0.55 = 0.45$$

Colour	Blue	Red	Green	Yellow
Probability	0.05	0.15	0.45	0.35

[2]

(b) Work out the probability that a counter taken at random is blue or yellow.

$$0.05 + 0.35 = 0.4$$

(b) 0.4 [2]

2 Solve these equations.

(a) $3x + 7 = 2x + 5$

$$3x - 2x = 5 - 7$$

$$x = -2$$

(a) $x = -2$ [2]

(b) $\frac{3x}{4} = 6$

$$3x = 6 \times 4$$

$$3x = 24$$

$$x = \frac{24}{3}$$

$$x = 8$$

(b) $x = 8$ [2]

(c) $5(2x + 1) = 20$

$$10x + 5 = 20$$

$$10x = 20 - 5$$

$$10x = 15$$

$$x = \frac{15}{10}$$

$$x = 1.5$$

(c) $x = 1.5$ [3]

- 3 (a) Show that $\frac{13}{50}$ is the same as 26%.

$$\frac{13}{50} = \frac{13}{50} \times \frac{2}{2} = \frac{26}{100} = 26\%$$

[1]

- (b) By writing each of these three fractions as percentages, arrange them in order, smallest first.

$$\frac{7}{20}$$

$$\frac{13}{50}$$

$$\frac{90}{300}$$

Show your working clearly.

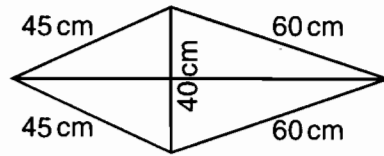
$$\frac{7}{20} = 0.35 = 35\%$$

$$\frac{13}{50} = 26\%$$

$$\frac{90}{300} = \frac{30}{100} = 30\%$$

(b) $\frac{13}{50}$ $\frac{90}{300}$ $\frac{7}{20}$ [3]
smallest

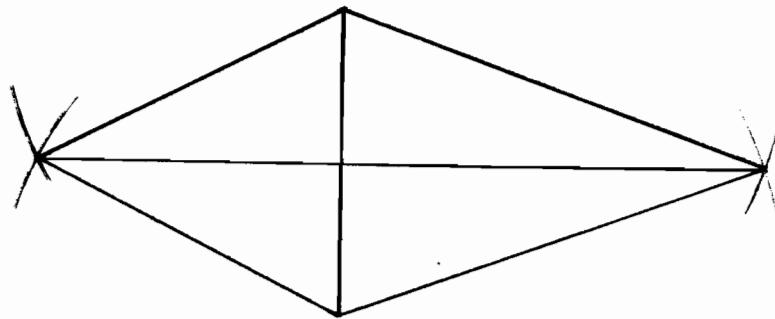
- 4 Here is a sketch of a kite.



NOT TO
SCALE

- (a) Using ruler and compasses, make an accurate scale drawing of the kite.
Use a scale of 1 cm to represent 10 cm.

[3]



- (b) Use your scale drawing to work out the length of the longer diagonal of the **real** kite.

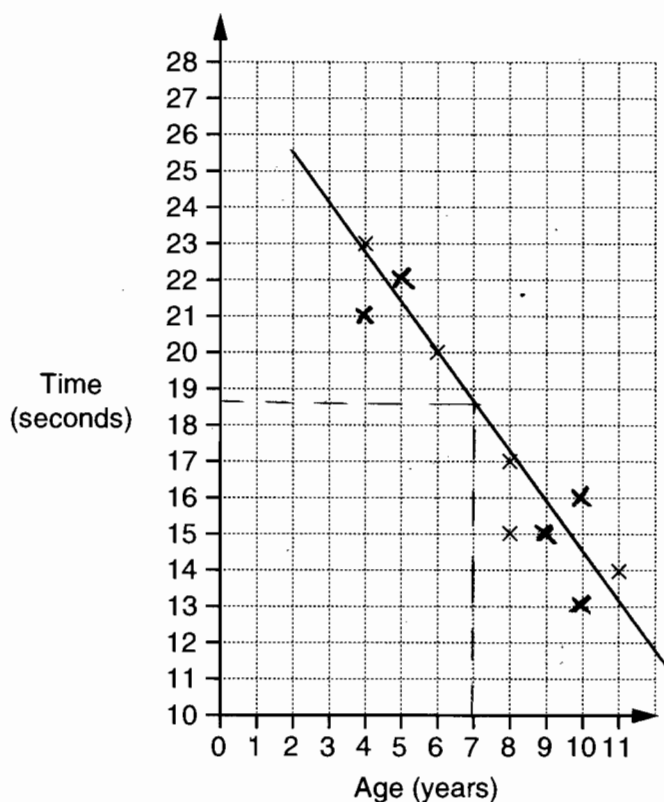
$$9.6 \text{ cm} \times 10 = 96 \text{ cm}$$

(b) 96 cm [2]

- 5 There are 10 children in a junior swimming club.
The table shows each child's age and their time to swim 30 metres.

Age (years)		10	5	4	9	10
Time (seconds)		13	22	21	15	16

- (a) Complete the scatter diagram.
The first 5 points have already been plotted.



[2]

- (b) Describe the correlation shown in the scatter diagram.

Negative Correlation

[1]

- (c) Draw a line of best fit on your diagram.

[1]

- (d) Hafisa, aged 7 years, joins the swimming club.

Use your line of best fit to estimate the time she will take to swim 30m.

(d) 18.5 s [1]

6 Ready salted crisps can be bought in

a pack of 6 bags for £1.38
or a pack of 10 bags for £2.20.

- (a) Which of these two packs is better value for money?
Show your working clearly.

Look at cost of 30 bags

5 lots of 6 £1.38

5 x

6.90

3 lots of 10 £2.20

3 x

£6.60

(a) 10 bags for £2.20 [3]
better value

- (b) A family pack contains only bags of smokey bacon crisps and bags of cheese and onion crisps.

The ratio of bags of smokey bacon to bags of cheese and onion is 3 : 2.

- (i) Phil says that each family pack contains 3 bags of smokey bacon crisps and 2 bags of cheese and onion crisps.

Explain why Phil may be wrong.

3:2 is ratio not actual numbers

could be 6 bacon and 4 cheese [1]

- (ii) Some family packs are opened and all the bags of crisps put into an empty container.
There are 160 bags of crisps altogether in the container.

How many bags of each flavour are there?

3 + 2 = 5 parts

5 parts = 160

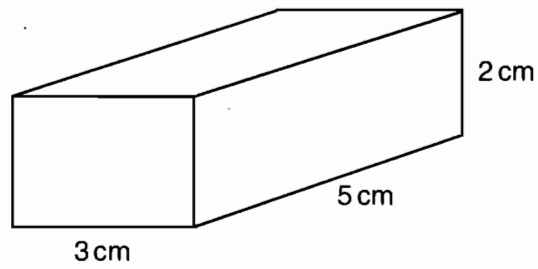
1 part = $\frac{160}{5} = 32$

bacon $3 \times 32 = 96$

cheese $2 \times 32 = 64$

(b)(ii) smokey bacon 96
cheese and onion 64 [3]

7



This cuboid has a mass of 135 g.

Work out the density of the cuboid.
Give the units of your answer.

$$\text{Volume} = 5 \times 3 \times 2 = 30 \text{ cm}^3$$

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} = \frac{135 \text{ g}}{30 \text{ cm}^3}$$

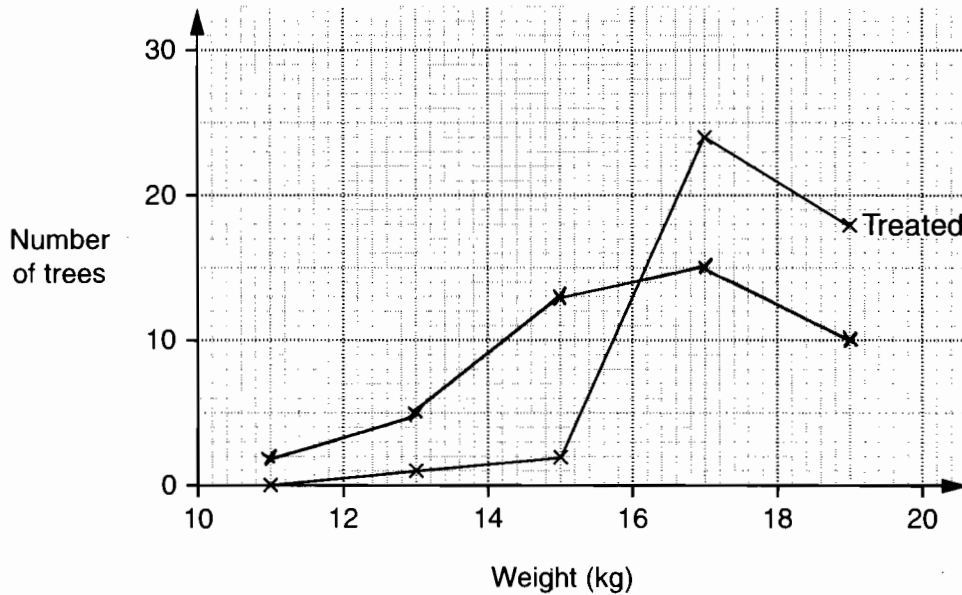
$$\frac{135}{30} = 13.5 \quad 3 \overline{)13.5}$$

$$\underline{4.5 \text{ g cm}^{-3}} \quad [4]$$

$$\text{or } 4.5 \text{ g/cm}^3$$

- 8 In an experiment on pest control and the production of fruit, 45 apple trees were treated with a pesticide and 45 other apple trees were left untreated. When the apples were picked, the total weight of apples from each tree was recorded.

The frequency polygon shows the distribution of weights of apples from the **treated** trees.



- (a) The table shows the distribution of weights of apples from the **untreated** trees.

Weight (wkg)	$10 < w \leq 12$	$12 < w \leq 14$	$14 < w \leq 16$	$16 < w \leq 18$	$18 < w \leq 20$
Number of trees	2	5	13	15	10

On the grid above, draw the frequency polygon for these data.

[2]

- (b) (i) Make one comment to compare the two distributions.

Treated trees give more weight

[1]

- (ii) Is it possible to decide which type of tree, treated or untreated, produces heavier individual apples?

Give a reason for your answer.

No because only total weight recorded
or amount of apples unknown

[1]

- 9 (a) Calculate an **estimate** of $\frac{3.5 \times 7.8}{0.46}$. $\approx \frac{4 \times 8}{0.5} = \frac{32}{0.5} = \frac{320}{5} = 64$
 Show clearly the values you use.

.....

(a) 64 [2]

- (b) Evaluate.

(i) $(\sqrt{5})^2 = \sqrt{5} \times \sqrt{5} = 5$

.....

(b)(i) 5 [1]

(ii) $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

.....

(ii) $\frac{1}{8}$ [1]

(iii) $4^0 = 1$

.....

(iii) 1 [1]

- 10 A ball is thrown into the air.
The height, h metres, of the ball above the ground after a time t seconds is given by

$$h = 25t - 5t^2.$$

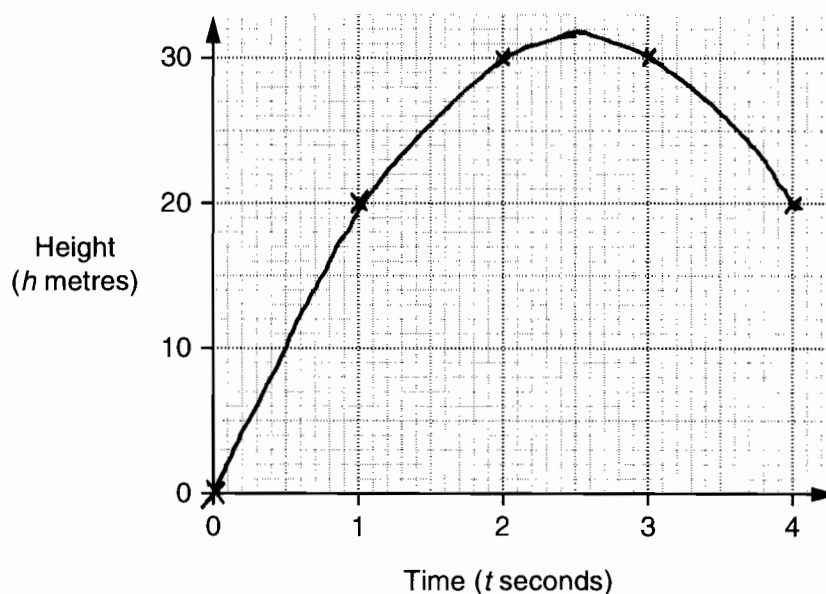
- (a) Complete the table of values.

$$\begin{array}{l} h = 25(1) - 5(1)^2 \\ = 25 - 5 \\ = 20 \end{array} \qquad \begin{array}{l} h = 25(2) - 5(2)^2 \\ = 50 - 20 \\ = 30 \end{array}$$

t	0	1	2	3	4
h	0	20	30	30	20

[2]

- (b) Draw the graph of $h = 25t - 5t^2$ for t from 0 to 4.



[2]

- (c) Use your graph to estimate

- (i) the maximum height of the ball above the ground,

(c)(i) 32 m [1]

- (ii) the time when the ball is 15 m above the ground.

(ii) 0.8 s [1]

11 A box of apples weighs 25 kg, correct to the nearest kilogram.

(a) Explain why the **upper bound** of the weight of 10 of these boxes of apples is 255 kg.

Upper bound for 1 box is 25.5 kg

$$25.5 \times 10 = 255 \text{ kg} \quad [1]$$

(b) 10 of these boxes of apples are put onto a pallet.
The pallet weighs 8 kg, correct to the nearest kilogram.

Work out the **least** possible total weight of the 10 boxes of apples and the pallet.

$$245 \text{ kg} + 7.5 \text{ kg} = 252.5 \text{ kg}$$

(b) 252.5 kg [2]

- 12 (a) Rearrange this formula to make x the subject.

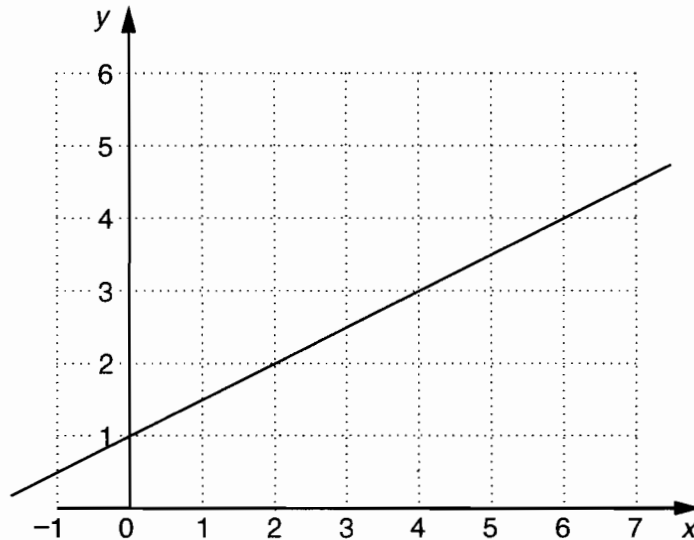
$$y = 3x - 2$$

$$y + 2 = 3x$$

$$\frac{y+2}{3} = x$$

(a) $x = \frac{y+2}{3}$ [2]

- (b)



- (i) Work out the gradient of this line.

Two points on line $(0, 1)$ and $(6, 4)$

$$\text{gradient} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 1}{6 - 0} = \frac{3}{6} = \frac{1}{2}$$

(b)(i) $\frac{1}{2}$ [2]

- (ii) Hence, write down the equation of the line.

y -intercept = 1 so $y = \frac{1}{2}x + 1$

(ii) $y = \frac{1}{2}x + 1$ [2]

- (c) Work out the coordinates of the midpoint of the line joining $(-3, 3)$ and $(4, 5)$.

$$\text{Midpoint} \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left(\frac{-3 + 4}{2}, \frac{3 + 5}{2} \right)$$

(c) $\left(\frac{1}{2}, 4 \right)$ [2]

- 13 The price of a cooker is reduced by 20% to £320.

Work out the price of the cooker **before** the reduction.

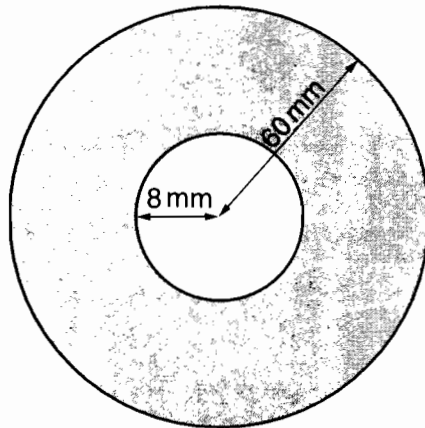
$$80\% \text{ of original} = \pounds 320$$

$$10\% \text{ of original} = \pounds 320 \div 8 = \pounds 40$$

$$100\% \text{ of original} = \pounds 40 \times 10 = \pounds 400$$

£ 400 [3]

- 14 A CD is a circular disc of radius 60 mm with a hole of radius 8 mm cut from the middle.



NOT TO
SCALE

Work out the area of one side of a CD, shown shaded.
Give your answer as a multiple of π .

$$\text{Area of circle} = \pi r^2$$

$$\text{Area shaded} = \pi \times 60^2 - \pi \times 8^2$$

$$= 3600\pi - 64\pi$$

$$\begin{array}{r} 5191 \\ 3600 \end{array}$$

$$\begin{array}{r} 64 \\ \hline 3536 \end{array}$$

$$3536$$

$$= 3536\pi \text{ mm}^2$$

$$\underline{3536\pi} \text{ mm}^2 [4]$$

16 (a) Solve.

$$(x+5)(x-1) = 0$$

Either $x+5=0 \Rightarrow x=-5$
 or $x-1=0 \Rightarrow x=1$

(a) $x=-5, x=1$ [1]

(b) Multiply out these brackets and simplify your answer.

$$(2y+3)(5y-2)$$

$$= 10y^2 + 15y - 4y - 6$$

$$= 10y^2 + 11y - 6$$

(b) $10y^2 + 11y - 6$ [3]

- 18 y is **inversely** proportional to the square of x .
When $y = 9$, $x = 2$.

(a) Use this information to find a formula connecting y and x .

$$y = \frac{k}{x^2} \quad y = 9, \text{ when } x = 2$$

$$9 = \frac{k}{2^2} \quad 9 = \frac{k}{4} \quad 9 \times 4 = k \quad k = 36$$

$$(a) \quad y = \frac{36}{x^2} \quad [2]$$

- (b) Find the value of y when $x = 10$. When $x = 10$ $y = \frac{36}{10^2} = \frac{36}{100}$

$$(b) \quad y = 0.36 \quad [1]$$

- (c) Find the values of x when $y = 4$.

$$\text{when } y = 4$$

$$4 = \frac{36}{x^2}$$

$$4x^2 = 36$$

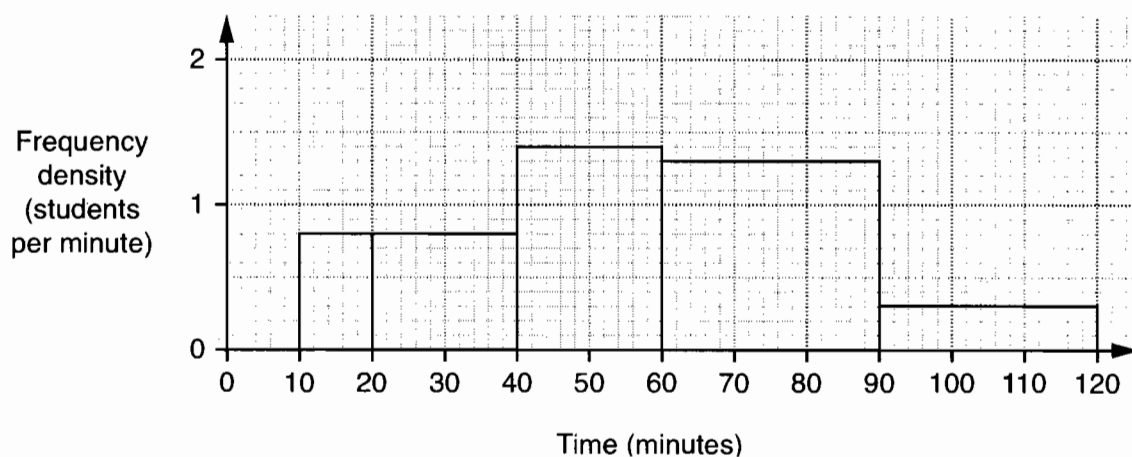
$$x^2 = \frac{36}{4} = 9$$

$$x = \pm \sqrt{9}$$

$$x = \pm 3$$

$$(c) \quad x = \pm 3 \quad [2]$$

- 19 Some Year 12 students were each asked how many minutes they exercised each day. The histogram shows the distribution of their times.



- (a) Josie says, "There are as many students exercising between 10 and 20 minutes as there are exercising between 20 and 40 minutes".

Explain why Josie is wrong.

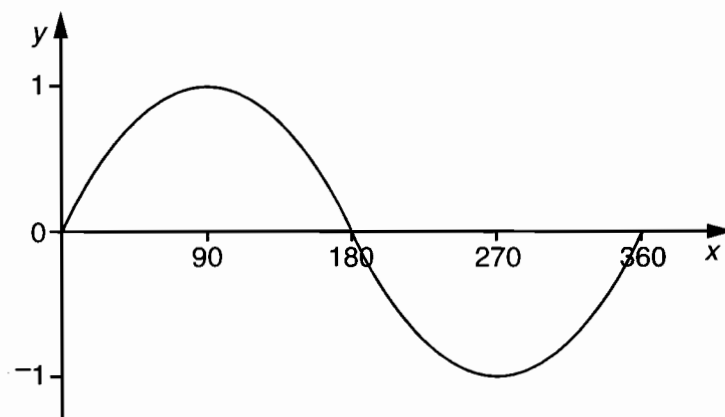
Frequency is proportional to area
so twice as many from 20-40 compared with 10-20 [1]

- (b) Calculate an estimate of the number of these students who exercised for 50 minutes or more.

$$\begin{array}{r}
 1.4 \times 10 + 1.3 \times 30 + 0.3 \times 30 \\
 \hline
 = 14 + 39 + 9 = 62 \\
 \hline
 \begin{array}{ccc}
 50-60 & 60-90 & 90-120
 \end{array}
 \end{array}$$

(b) 62 [3]

20 Here is the graph of $y = \sin x^\circ$ for $0 \leq x \leq 360$.



One solution of $\sin x^\circ = 0.5$ is $x = 30$.

(a) Find another solution of $\sin x^\circ = 0.5$ for $0 \leq x \leq 360$.

.....

(a) 150° [1]

(b) Find the solutions of $\sin x^\circ = -0.5$ for $0 \leq x \leq 360$.

.....

(b) 210°, 330° [2]

21 Solve algebraically these simultaneous equations.

$$\begin{aligned}x^2 + y^2 &= 17 \\ y &= x - 3\end{aligned}$$

①
②

Subst for y in ①

$$x^2 + (x - 3)^2 = 17$$

$$x^2 + x^2 - 6x + 9 = 17$$

$$2x^2 - 6x - 8 = 0$$

$$x^2 - 3x - 4 = 0$$

$$(x - 4)(x + 1) = 0$$

$$\Rightarrow x - 4 = 0 \quad \text{or} \quad x + 1 = 0$$

$$x = 4$$

$$x = -1$$

when $x = 4$

$$y = 4 - 3 = 1$$

when $x = -1$

$$y = -1 - 3 = -4$$

$$x = 4, y = 1$$

$$x = -1, y = -4$$

[6]