

A\* 79    A 63    B 47    C 32



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**GENERAL CERTIFICATE IN SECONDARY EDUCATION  
MATHEMATICS SYLLABUS A**

**J512/03**

Paper 3  
(Higher Tier)

Solutions

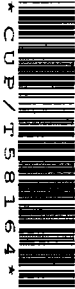
Candidates answer on the question paper

**OCR Supplied Materials:**  
None

**Other Materials Required:**  
• Geometrical instruments  
• Tracing paper (optional)

**Friday 9 January 2009  
Morning**

**Duration: 2 hours**



<b>Candidate Forename</b>		<b>Candidate Surname</b>	
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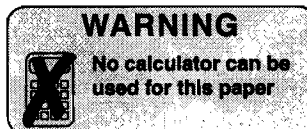
<b>Centre Number</b>										<b>Candidate Number</b>				
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**INSTRUCTIONS TO CANDIDATES**

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the boxes above.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Show your working. Marks may be given for a correct method even if the answer is incorrect.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Write your answer to each question in the space provided, however additional paper may be used if necessary.

**INFORMATION FOR CANDIDATES**

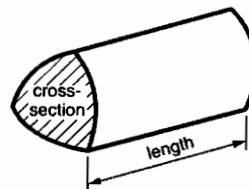
- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is **100**.
- This document consists of **20** pages. Any blank pages are indicated.



<b>FOR EXAMINER'S USE</b>

## Formulae Sheet: Higher Tier

**Volume of prism** = (area of cross-section)  $\times$  length

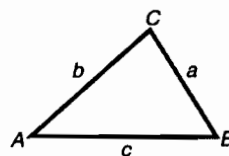


**In any triangle ABC**

**Sine rule**  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

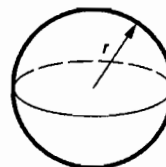
**Cosine rule**  $a^2 = b^2 + c^2 - 2bc \cos A$

**Area of triangle** =  $\frac{1}{2} ab \sin C$



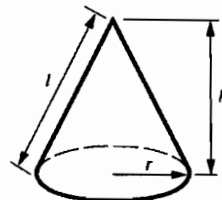
**Volume of sphere** =  $\frac{4}{3} \pi r^3$

**Surface area of sphere** =  $4\pi r^2$



**Volume of cone** =  $\frac{1}{3} \pi r^2 h$

**Curved surface area of cone** =  $\pi r l$



**The Quadratic Equation**

The solutions of  $ax^2 + bx + c = 0$ ,  
where  $a \neq 0$ , are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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1 Linda works in a sandwich factory.

(a) She makes 30 sandwiches every hour.

How long does it take her to make 220 sandwiches?  
Give your answer in hours and minutes.

$$\frac{220}{30} = \frac{22}{3} = 7\frac{1}{3} \text{ hours}$$

(a) 7 hours 20 minutes [3]

(b) Linda's wage is £360 a week.  
She receives a 5% wage rise.

$$10\% = £36$$

Work out Linda's new weekly wage.

$$5\% = £18$$

$$\text{New wage } £360 + £18 = £378$$

(b) £ 378 [3]

(c) Linda makes cheese sandwiches and chicken sandwiches in the ratio 2 : 3.  
She makes 200 sandwiches altogether.

How many of these are cheese sandwiches?

$$2 + 3 = 5 \text{ parts}$$

$$1 \text{ part} = 200 \div 5 = 40$$

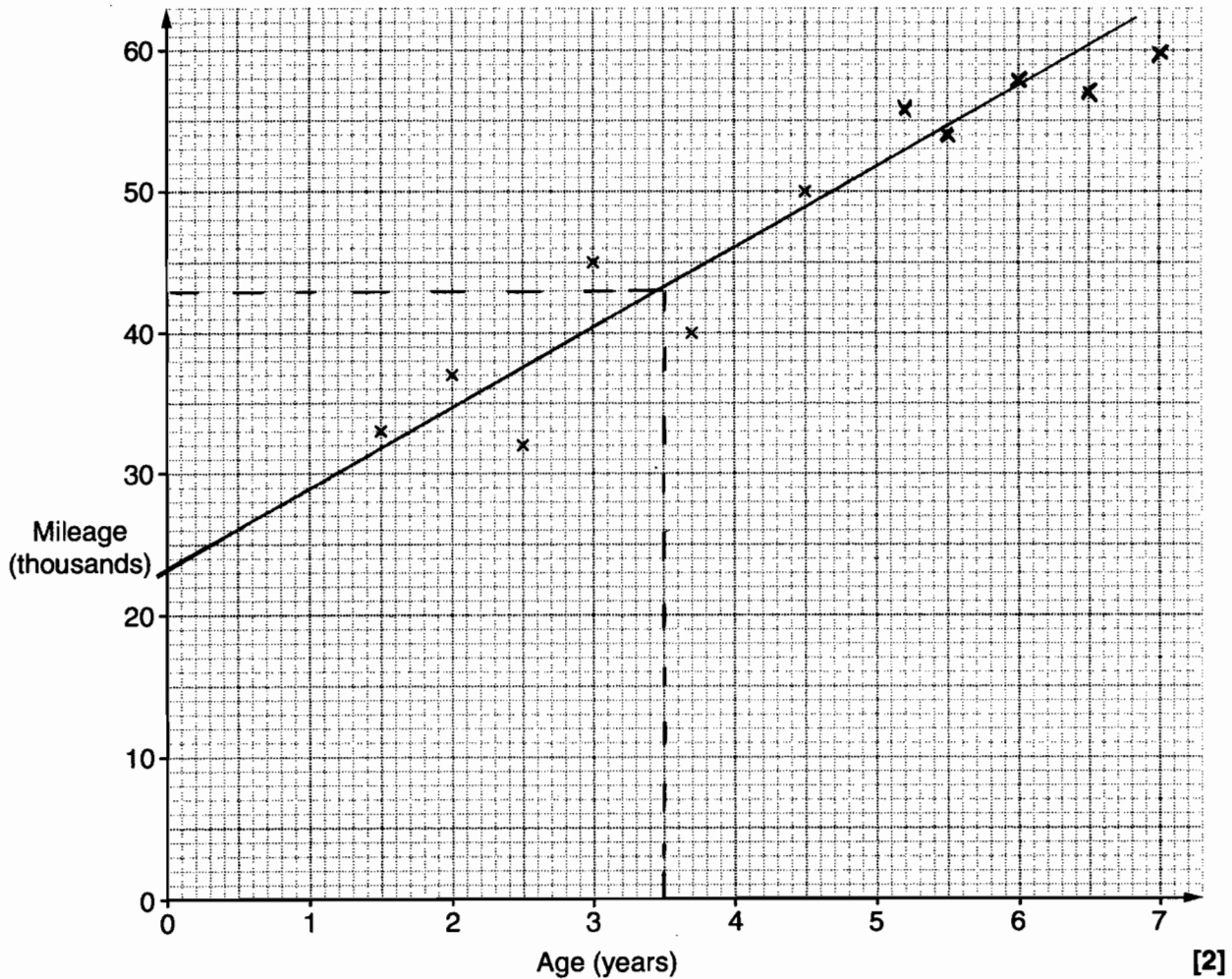
$$\text{cheese } 2 \text{ parts} = 40 \times 2 = 80$$

(c) 80 [2]

2 As part of a project, Robert records the ages and mileages of some cars. His results are recorded in this table.

Age (years)	1.5	2	2.5	3	3.7	4.5	5.2	5.5	6	6.5	7
Mileage (thousands)	33	37	32	45	40	50	56	54	58	57	60

(a) Complete the scatter diagram. The first 6 points have already been plotted.



(b) Describe the strength and type of correlation shown in your diagram.

(b) strong positive [2]

(c) (i) Draw a line of best fit for these data. [1]

(ii) Another car is 3.5 years old.

Use your line of best fit to estimate the mileage of this car.

(c)(ii) 43 thousand miles [1]

3 (a)  $P = 5x - 2y$

Work out the value of  $P$  when  $x = 3$  and  $y = -4$ .

$$P = 5(3) - 2(-4)$$

$$= 15 + 8 = 23$$

(a) 23 [2]

(b)  $Q = 2x + 5$

Work out the value of  $x$  when  $Q = 13$ .

$$13 = 2x + 5$$

$$13 - 5 = 2x$$

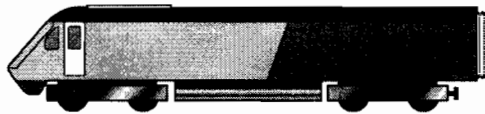
$$8 = 2x$$

$$\frac{8}{2} = x$$

$$4 = x$$

(b)  $x = 4$  [2]

4 A model of a railway engine is made to a scale of 2 cm to 1 m.



(a) The length of the railway engine is 24 metres.

Work out the length of the model.

$$24 \times 2 \text{ cm} = 48 \text{ cm}$$

(a) 48 cm [2]

(b) The height of the model is 8 cm.

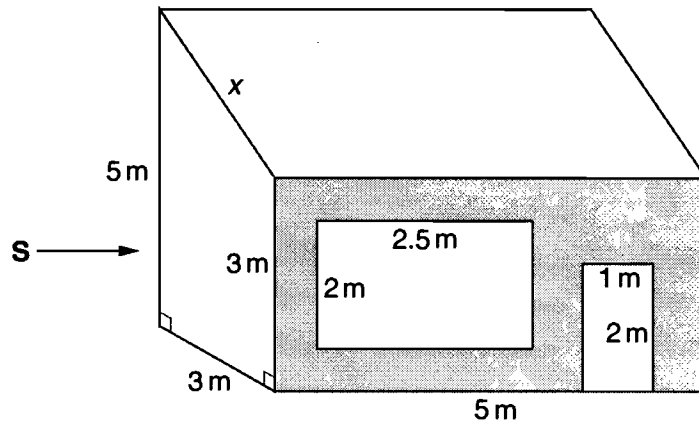
Work out the height of the railway engine.

$$8 \text{ cm} = 4 \times 2 \text{ cm}$$

$$4 \times 1 \text{ m} = 4 \text{ m}$$

(b) 4 m [2]

- 5 The diagram shows a small shop.  
The front of the shop, the window and the door are all rectangles.



- (a) Work out the shaded area of the front of the shop.  
Give the units of your answer.

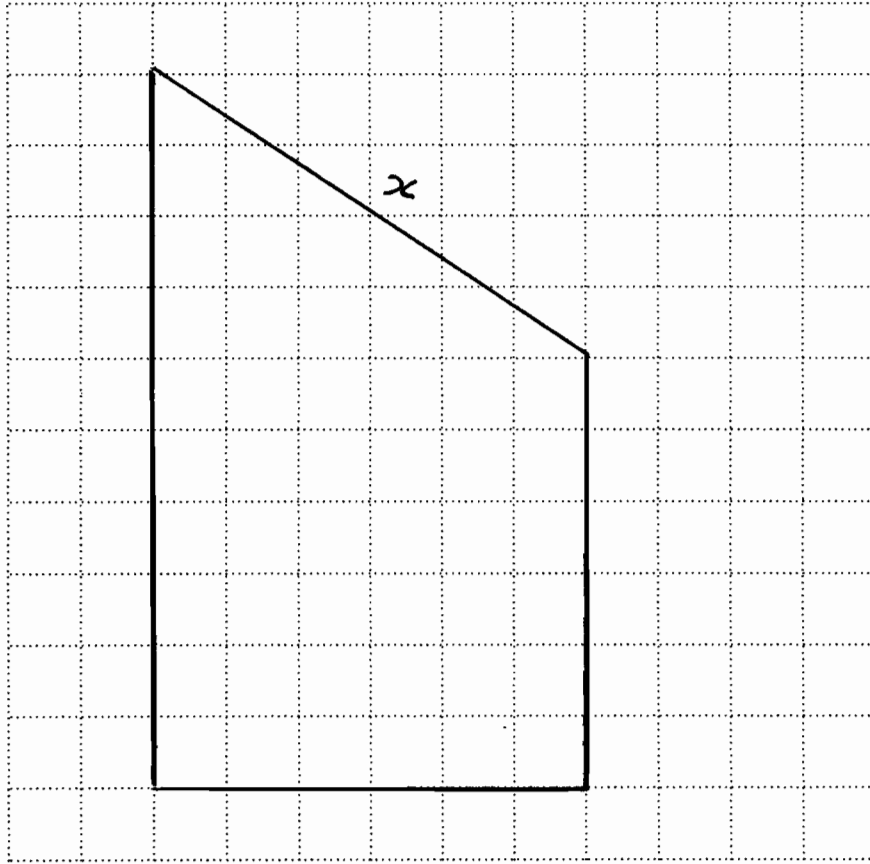
$$\text{Window } 2 \times 2.5 = 5 \text{ m}^2 \quad \text{door } 2 \times 1 = 2 \text{ m}^2$$

$$\text{Shaded area} = 5 \times 3 - 5 - 2$$

$$= 15 - 5 - 2 = 8 \text{ m}^2$$

(a) 8 m<sup>2</sup> [3]

- (b) On the centimetre grid, draw the side elevation of the shop (the view from S).  
Use a scale of 2 cm for 1 m.



[2]

- (c) Use your drawing in part (b) to find the width of the roof,  $x$ .

(c) 3.5 m [1]

$$x = 7 \text{ cm on drawing}$$

$$2 \text{ cm represents } 1 \text{ m}$$

$$1 \text{ cm represents } 0.5 \text{ m}$$

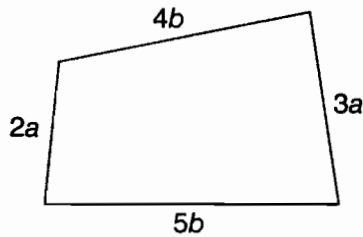
$$7 \times 0.5 = 3.5 \text{ m}$$

6 (a) Simplify.

$$2xy - 3xy + 4xy$$

.....  
 .....  
 (a) 3xy [1]

(b) Find an expression for the perimeter of this shape.  
 Give your answer as simply as possible in terms of  $a$  and  $b$ .



.....  
 Perimeter =  $2a + 4b + 3a + 5b$   
 =  $5a + 9b$   
 .....

(b) 5a + 9b [2]

(c) Multiply out and simplify.

$$3(2x + 5) + 2(4x - 1)$$

.....  
 =  $6x + 15 + 8x - 2$   
 =  $14x + 13$   
 .....

(c) 14x + 13 [2]



- 7 Estimate the answer to this calculation.

$$\frac{112 \times 5.8}{0.47} \approx \frac{100 \times 6}{0.5}$$

Show clearly the values you use.

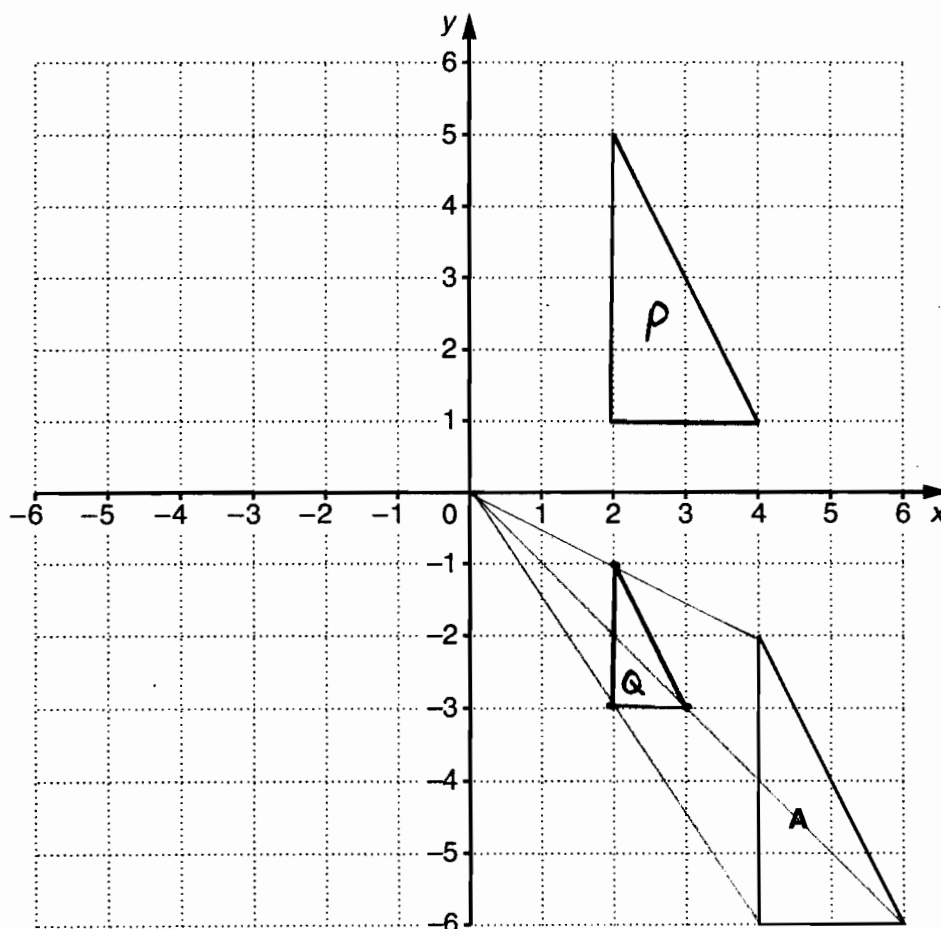
$$= \frac{600}{0.5} = \frac{6000}{5}$$

$$= 1200$$

$$\underline{1200}$$

[2]

8



- (a) Translate triangle **A** by  $\begin{pmatrix} -2 \\ 7 \end{pmatrix}$ .

Label the image **P**.

[2]

- (b) Enlarge triangle **A** by scale factor  $\frac{1}{2}$  using centre  $(0, 0)$ .

Label the image **Q**.

[2]

- 9 (a) Show that the equation  $x^3 - 10x + 7 = 0$  has a solution between 2 and 3.

$$\begin{array}{l} x = 2 \quad 2^3 - 10(2) + 7 = 8 - 20 + 7 = -5 \\ x = 3 \quad 3^3 - 10(3) + 7 = 27 - 30 + 7 = +4 \end{array}$$

A sign change between  $x = 2$  and  $x = 3$  indicates  
expression = 0 for some value between  $x = 2$  and  $x = 3$  [3]

- (b) Solve.

$$3(2x - 1) > 12$$

$$\begin{array}{l} \dots\dots\dots \\ 6x - 3 > 12 \\ \dots\dots\dots \\ 6x > 12 + 3 \\ \dots\dots\dots \\ 6x > 15 \\ \dots\dots\dots \\ x > \frac{15}{6} \\ \dots\dots\dots \\ x > 2\frac{1}{2} \end{array}$$

(b)  $x > 2\frac{1}{2}$  [3]

- 10 (a) Write each of the following in standard form.

(i) 455000

(a)(i)  $4.55 \times 10^5$  [1]

(ii) 0.000038

(ii)  $3.8 \times 10^{-5}$  [1]

(iii)  $29 \times 10^8$

(iii)  $2.9 \times 10^9$  [1]

- (b) Work out.

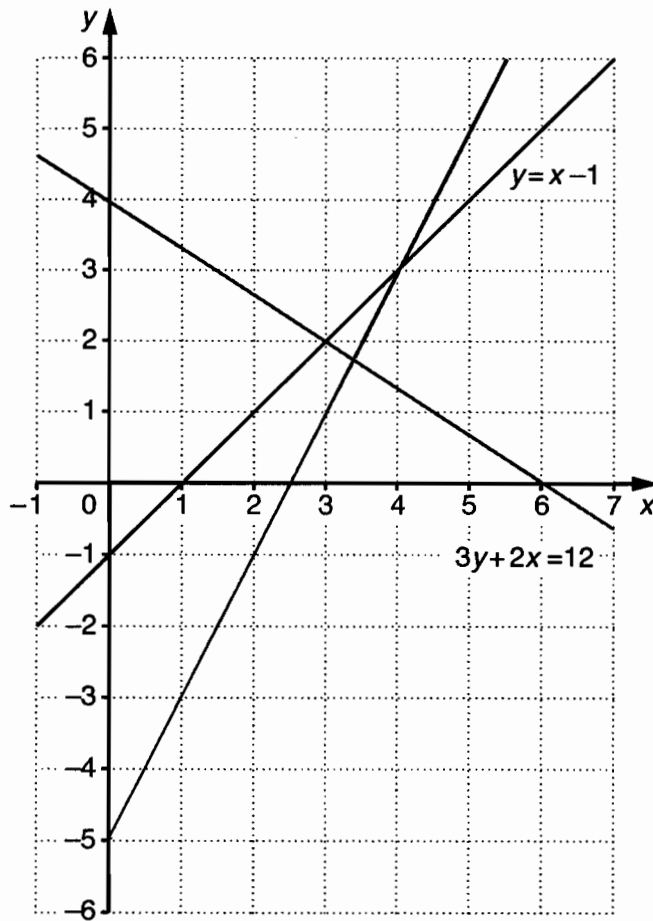
$$\frac{8 \times 10^{12}}{2 \times 10^3} = 4 \times 10^9$$

Give your answer in standard form.

This comes from cancelling  $8 \div 2 = 4$   
 $10^{12} \div 10^3 = 10^9$

(b)  $4 \times 10^9$  [2]





(a) Use the diagram to solve these simultaneous equations.

$$\begin{aligned} 3y + 2x &= 12 \\ y &= x - 1 \end{aligned}$$

(a)  $x = \underline{3}$   $y = \underline{2}$  [1]

(b) By drawing another straight line on the diagram, solve these simultaneous equations.

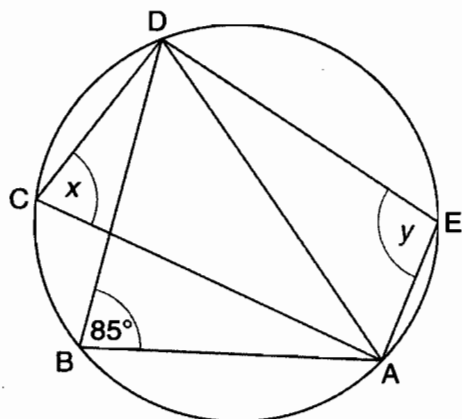
$$\begin{aligned} y &= x - 1 \\ y &= 2x - 5 \end{aligned}$$

Draw  $y = 2x - 5$

$x$	0	1	2	5
$y$	-5	-3	-1	5

(b)  $x = \underline{4}$   $y = \underline{3}$  [3]

- 13 A, B, C, D and E are points on the circumference of a circle.  
Angle ABD =  $85^\circ$ .



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- (a) Explain why AD is **not** a diameter of the circle.

Because  $\angle ABD$  would then be  $90^\circ$ , an angle  
in a semi-circle. [1]

- (b) Find angle x.  
Give a reason for your answer.

$x = 85^\circ$  because angles in same segment are  
equal [2]

- (c) Work out angle y.  
Give a reason for your answer.

$y = 95^\circ$  because opposite angles of a  
cyclic quadrilateral add up to  $180^\circ$  [2]

14 A shopkeeper recorded the amount of money spent by each of 100 customers. This table summarises the data.

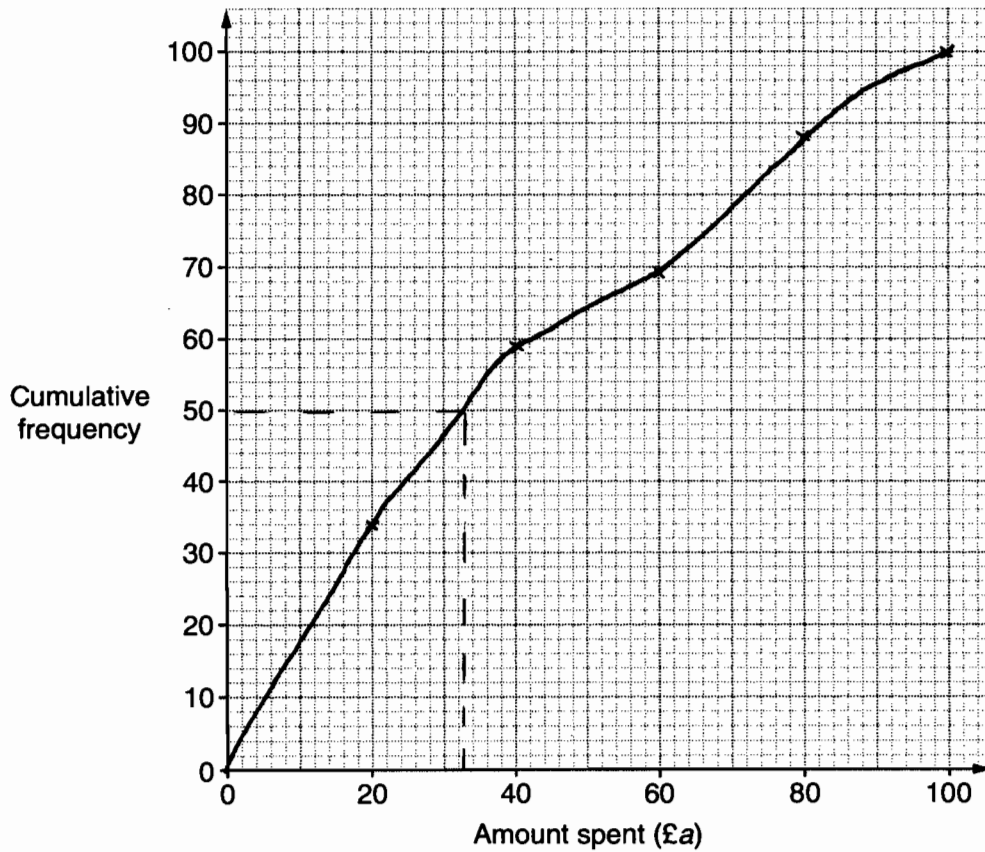
Amount spent (£ $a$ )	$0 < a \leq 20$	$20 < a \leq 40$	$40 < a \leq 60$	$60 < a \leq 80$	$80 < a \leq 100$
Number of customers (frequency)	34	25	10	19	12

(a) Complete the cumulative frequency table.

Amount spent (£ $a$ )	$a \leq 20$	$a \leq 40$	$a \leq 60$	$a \leq 80$	$a \leq 100$
Cumulative frequency	34	59	69	88	100

[1]

(b) On this grid, draw a cumulative frequency graph for these data.



[3]

(c) Use your graph to find the median amount spent.

(c) £ 33 [1]

15 (a) (i) Factorise.

$x^2 + 7x + 12$

$(x+3)(x+4)$

$(a)(i) \underline{(x+3)(x+4)} \quad [2]$

(ii) Hence, solve this equation.

$x^2 + 7x + 12 = 0$

$(x+3)(x+4) = 0$

$E, \text{ther } x+3 = 0$

$\text{or } x+4 = 0$

$\Rightarrow x = -3$

$\Rightarrow x = -4$

$(ii) \underline{x = -3, x = -4} \quad [1]$

(b) Factorise.

$x^2 - 4y^2$

$(x+2y)(x-2y)$

Difference of two squares

$(b) \underline{(x+2y)(x-2y)} \quad [2]$

(c) Expand and simplify.

$(3x+2)(2x-5)$

$= 6x^2 + 4x - 15x - 10$

$= 6x^2 - 11x - 10$

$(c) \underline{6x^2 - 11x - 10} \quad [3]$

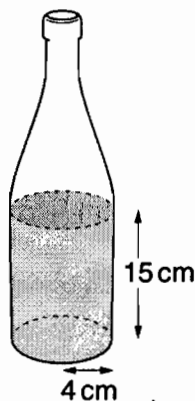




- 18 The hemispherical bowl of a small glass has internal radius 3 cm.



The lower part of a bottle is a cylinder of internal radius 4 cm. It contains fruit juice to a depth of 15 cm.



Sphere Vol

$$\frac{4}{3} \pi r^3$$

hemisphere  $\frac{2}{3} \pi r^3$

$$= \frac{2}{3} \pi \times 3^3$$

$$= 18\pi \text{ cm}^3$$

Cylinder Vol

$$\pi r^2 h$$

$$= \pi \times 4^2 \times 15$$

How many of these glasses can be completely filled using all the fruit juice in this bottle?  
Leave  $\pi$  in your calculations.

$$= \pi \times 16 \times 15 = 240\pi \text{ cm}^3$$

$$\text{Number of glasses filled} = \frac{240\pi}{18\pi}$$

$$= \frac{240}{18} = \frac{40}{3}$$

$$= 13\frac{1}{3}$$

Completely filled glasses is therefore 13

13

[5]

- 19 In an experiment, the temperature of a liquid is measured as it cools. This is the formula that gives the temperature,  $T$  °C, of the liquid  $m$  minutes after the start of the experiment.

$$T = 60 \times 2^{-m} + 25$$

- (a) Work out the temperature of the liquid at the start of the experiment.

At start  $m = 0$

$$T = 60 \times 2^0 + 25$$

$$= 60 \times 1 + 25 = 85^\circ$$

(a) 85 °C [2]

- (b) Work out the temperature of the liquid 2 minutes after the start of the experiment.

When  $m = 2$

$$T = 60 \times 2^{-2} + 25$$

$$= 60 \times \frac{1}{2^2} + 25$$

$$= \frac{60}{4} + 25 = 15 + 25 = 40$$

(b) 40 °C [2]

20 Solve algebraically these simultaneous equations.

$$\begin{aligned} 3x + 2y &= 7 \\ y &= x^2 - 2x + 3 \end{aligned}$$

①  
②

$$\textcircled{2} \times 2$$

$$2y = 2x^2 - 4x + 6$$

③

From ①

$$2y = 7 - 3x$$

④

Subst for  $2y$  in ③

$$7 - 3x = 2x^2 - 4x + 6$$

$$0 = 2x^2 - 4x + 6 + 3x - 7$$

$$0 = 2x^2 - x - 1$$

$$0 = (2x + 1)(x - 1)$$

$$\Rightarrow 2x + 1 = 0 \quad \text{or} \quad x - 1 = 0$$

$$2x = -1$$

$$x = +1$$

$$x = -\frac{1}{2}$$

[7]

Sub for  $x$  in ④

$$2y = 7 - 3\left(-\frac{1}{2}\right)$$

$$2y = 7 + \frac{3}{2}$$

$$2y = \frac{17}{2}$$

$$y = \frac{17}{4}$$

$$\text{or} \quad 2y = 7 - 3(1)$$

$$2y = 7 - 3$$

$$2y = 4$$

$$y = \frac{4}{2}$$

$$y = 2$$

Solution

$$\begin{cases} x = -\frac{1}{2} \\ y = \frac{17}{4} \end{cases}$$

$$\begin{cases} x = 1 \\ y = 2 \end{cases}$$

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